

URGANCH DAVLAT UNIVERSITETI
HUZURIDAGI ILMIY DARAJA BERUVCHI
PhD.03/30.12.2019.FM.55.01 RAQAMLI ILMIY KENGASH

URGANCH DAVLAT UNIVERSITETI

RAXIMOV ILXAM DAVRONBEKOVICH

**YUKLANGAN NOCHIZIQLI EVOLYUTION TENGLAMALARNI
YECHISHNING TO‘G‘RI VA TESKARI USULLARI**

01.01.02 – Differensial tenglamalar va matematik fizika

**FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD)
DISSERTATSIYASI AVTOREFERATI**

Urganch – 2022

**Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi
avtoreferati mundarijasi**

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(PhD) on physical-mathematical sciences**

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zaruriyati. Jahon miqyosida olib borilayotgan ko‘plab ilmiy-amaliy tadqiqotlar, ba’zi hollarda xususiy hosilali nochiziqli differensial tenglamalarni ifodalovchi matematik modellar bilan tavsiflanadi. Ushbu turdagi nochiziqli differensial tenglamalar orasida maxsus yechimlarga ega tenglamalar sinfi, solitonlar alohida ajralib turadi. Solitonlar nazariyasida nochiziqli tenglamalar juda muhim o‘rin kasb etadi, masalan, Korteveg-de Friz tenglamasi, nochiziqli Shredinger tenglamasi, modifitsirlangan Korteveg-de Friz tenglamasi, shular jumlasidandir. Bu kabi tenglamalar kvant mexanikasi, nochiziqli optika, chuqur suvdagi to‘lqinlar nazariyasini o‘rganishda muhim rol o‘ynaydi. Shu boisdan, nochiziqli Shredinger tenglamasi, Korteveg-de Friz tenglamasi va modifitsirlangan Korteveg-de Friz tenglamasini integrallash masalasi zamonaviy matematik fizikaning muhim vazifalaridan biri bo‘lib kelmoqda.

Hozirgi kunda jahonda nochiziqli to‘lqinlar nazariyasidagi izlanishlar, jumladan, egiluvchan trubalarda suyuqliklarning harakatlari, optik solitonlar telekommunikatsion texnologiyalar sohasida qo‘llanila boshladi. Haqiqiy fizik sistemalar klassik tenglamalar modifikatsiyalaridan biri bo‘lgan moslangan manbali tenglamalar bilan xarakterlanadi. Bundan tashqari, fizik sistemalarga ta’sir etuvchi kuchlar faqat vaqtning ma’lum bir davri mobaynida chegaralangan bo‘ladi, shuning uchun, haqiqiy modellar fazoviy o‘zgaruvchilar bo‘yicha ma’lum bir funksiyalar sinfidagi tenglamalarni o‘rganishga keltiriladi. Nochiziqli optika, plazma fizikasi, gidrodinamika va boshqa sohalarda moslangan manbali va yuklangan nochiziqli evolyutsion tenglamalarni o‘rganish fanning ustuvor yo‘nalishlaridan biri hisoblanadi. Shu munosabat bilan moslangan manbali va yuklangan nochiziqli Shredinger tenglamasi va yuklangan Korteveg-de Friz tenglamasi va modifitsirlangan Korteveg-de Friz tenglamasini o‘rganish maqsadli ilmiy tadqiqotlardan biri hisoblanadi.

Mamlakatimizda ilmiy va amaliy tatbiqqa ega bo‘lgan fundamental fanlarga alohida e’tibor qaratilmoqda. Xususan, nochiziqli to‘lqinlar nazariyasi, differensial operatorlarning spektral nazariyasi va matematik fizika masalalarini o‘rganishga e’tibor kuchaydi. Buning natijasida to‘g‘ri va teskari masalalar usuli yordamida matematik fizikaning nochiziqli evolyutsion tenglamalarini to‘la integrallashda salmoqli natijalarga erishildi. Matematik fizika va ushbu sohaning zamonaviy usullari sohasida xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish asosiy vazifa va yo‘nalish etib belgilandi¹. Ushbu qarorlar ijrosini ta’minlash maqsadida sochilish nazariyasining to‘g‘ri va teskari masala usuli va to‘g‘ri usullar yordamida moslangan manbali nochiziqli tenglamalarni, shuningdek, yuklangan nochiziqli tenglamalarni integrallashni rivojlantirish muhim ahamiyatga ega.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PF-4947-son “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi

¹ O‘zbekiston Respublikasi Vazirlar Mahkamasining 2017-yil 18-maydagi “O‘zbekiston Respublikasi Fanlar Akademiyasining yangidan tashkil etilgan ilmiy tadqiqot muassasalari faoliyatini tashkil etish to‘g‘risida”gi 292-sonli qarori.

to'g'risida", 2022-yil 28-yanvardagi PF-60-sonli "2022-2026 yillarga mo'ljallangan Yangi O'zbekistonning taraqqiyot strategiyasi to'g'risida"gi farmonlari, 2017-yil 17-fevraldagi PQ-2789-son "Fanlar akademiyasi faoliyati, ilmiy tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to'g'risida", 2017-yil 20-apreldagi PQ-2909-son "Oliy ta'lim tizimini yanada rivojlantirish chora-tadbirlari to'g'risida", 2018-yil 27-apreldagi PQ-3682-son "Innovasion g'oyalar, texnologiyalar va loyihalarni amaliyotga joriy qilish tizimini yanada takomillashtirish chora-tadbirlari to'g'risida", 2020-yil 7-maydagi PQ-4708-son "Matematika sohasidagi ta'lim sifatini oshirish va ilmiy tadqiqotlarni rivojlantirish chora-tadbirlari to'g'risida"gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalar rivojlanishining ustuvor yo'nalishlariga bog'liqligi. Ushbu tadqiqot O'zbekiston Respublikasi fan va texnologiyalar rivojlanishining IV. "Matematika, mexanika va informatika" ustuvor yo'nalishi doirasida bajarildi.

Muammoning o'rganilganlik darajasi. 1967 yilda C.S. Gardner, J.M. Grin, M. Kruskal va R. Miura tomonidan nochiziqli Korteveg-de Vries tenglamasi uchun qo'yilgan Koshi masalasining yechimi teskari masala usulidan foydalanib topilgan. P. Laks sochilish nazariyasining teskari masalasi usuli universal xarakterga ega ekanligini ko'rsatgan. Bu borada keyingi muhim natija 1972 yil V.E. Zaxarov va A.B. Shabat nochiziqli Shredinger tenglamasiga qo'yilgan Koshi masalasini Dirak operatori uchun sochilish nazariyasining to'g'ri va teskari masala usulidan foydalanib integrallashi bo'ldi. V.E.Zaxarov va A.B.Shabat ishlarining g'oyasidan foydalanib, M.Vadati modifitsirlangan Korteveg-de Vries tenglamasini yechishga muvaffaq bo'lgan. V.K. Melnikov nochiziqli evolyutsion tenglamalarga moslangan manba tushunchasini kiritadi va 1988 yilda moslangan manbali Korteveg-de Vries tenglamasi teskari masala usulidan foydalanib integrallash algoritmini ko'rsatib beradi.

Shuni ham ta'kidlash kerakki, J. Leon va A. Latifiyning ishlarida moslangan manbali Korteveg-de Friz tenglamasining fizik ma'nolari keltirilgan. A.B. Xasanov va G.U. Urazboev moslangan manbali Korteveg-de Friz tenglamasi, modifitsirlangan Korteveg-de Friz tenglamasi, nochiziqli Shredinger tenglamalarini sin-Gordon "tez kamayuvchi" funksiyalar sinfida sochilish nazariyasining teskari masala usulidan foydalanib integrallaydilar. A.B. Xasanov, A.A. Reyimberganov va boshqalarning ishlarida esa moslangan manbali Shredinger tipidagi tenglamalar o'rganilgan.

Bundan tashqari, R. Hirota tomonidan Korteveg-de Friz tenglamasi, modifitsirlangan Korteveg-de Friz va nochiziqli Shredinger tenglamalarining soliton yechimlarini topish algoritmi berilgan. Hirota to'g'ri usuli yordamida Y. L. Sun, W.X. Ma, J.P. Yu, A. M. Wazwaz, Y. Ye, L. Wang, Z. Chang, J. He, M. Gurses, A. Pekcan, W.X. Ma, S. Kumar, B. Mohan, Y. Zhang, L. Jin, C.T. Lee, C.C. Lee, F.You, J. Zhang, L. Li, C. Duan, F. Yular nochiziqli evolyutsion tenglamalarning aniq yechimlarini topganlar.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta'lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog'liqligi. Dissertatsiya tadqiqoti Urganch davlat universiteti ilmiy tadqiqot ishlari rejasiga muvofiq F-4-61 "Moslangan manbali noxiziqli evolyutsion tenglamalarni teskari masala usulida integrallash" (2012-2016 yy.) ilmiy tadqiqot loyihasi doirasida bajarilgan.

Tadqiqotning maqsadi differensial operatorlar uchun sochilish nazariyasining teskari masala yordamida tez kamayuvchi funksiyalar sinfida yuklangan noxiziqli Shredinger tenglamasining va to'g'ri usullar yordamida moslangan manbali noxiziqli Shredinger tenglamasi, yuklangan Korteveg-de Friz tenglamasi, yuklangan modifisirlangan Korteveg-de Friz tenglamasining yechimlarini topishdan iborat.

Tadqiqotning vazifalari:

tez kamayuvchi funksiyalar sinfida yuklangan noxiziqli Shredinger tenglamasiga qo'yilgan Koshi masalasining to'la integrallanuvchi bo'lishini isbotlash;

tez kamayuvchi funksiyalar sinfida yuklangan noxiziqli Shredinger tenglamasiga qo'yilgan Koshi masalasining yechimining yagonaligini isbotlash;

to'g'ri usullar yordamida tez kamayuvchi funksiyalar sinfida moslangan manbali noxiziqli Shredinger tenglamasining aniq yechimlarini topish;

to'g'ri usullar yordamida tez kamayuvchi funksiyalar sinfida yuklangan noxiziqli Shredinger tenglamasining aniq yechimlarini topish;

to'g'ri usullar yordamida yuklangan Korteveg-de Friz tenglamasining aniq yechimlarni topish;

to'g'ri usullar yordamida yuklangan modifisirlangan Korteveg-de Friz tenglamasining aniq yechimlarni topish.

Tadqiqotning ob'ekti yuklangan noxiziqli Shredinger tenglamasi, moslangan manbali noxiziqli Shredinger tenglamasi, yuklangan Korteveg-de Friz, yuklangan modifisirlangan Korteveg-de Friz tenglamalaridan iborat.

Tadqiqotning predmeti Dirak operatori uchun sochilish nazariyasining teskari masalasini, Hirota to'g'ri usulini va (G' / G) - usulini noxiziqli evolyutsion tenglamalarni integrallashga qo'llash usullaridan iborat.

Tadqiqotning usullari. Dissertatsiyada matematik analiz, matematik fizika, differensial operatorlarning spektral nazariyasi, funktsional analiz va kompleks o'zgaruvchili funksiyalar nazariyasidan, shuningdek, differensial tenglamalarni yechish usullaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

tez kamayuvchi funksiyalar sinfida yuklangan noxiziqli Shredinger tenglamasiga qo'yilgan Koshi masalasi yechimining yagonaligi va to'la integrallanuvchi bo'lishi isbotlangan;

sochilish nazariyasining asosiy integral tenglamasining sonli yechimlari topilgan;

Hirota to'g'ri usuli yordamida moslangan manbali va yuklangan noxiziqli Shredinger tenglamalarining soliton yechimlari topilgan;

yuklangan Korteveg-de Friz tenglamasi va modifisirlangan Korteveg-de Friz tenglamasining aniq yechimlari topilgan.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

yuklangan noxiziqli Shredinger tenglamasiga qo'yilgan Koshi masalasini, moslangan manbali noxiziqli Shredinger tenglamasi, yuklangan Korteveg-de Friz va yuklangan modifisirlangan Korteveg-de Friz tenglamasining yechimlarini topish algoritmlari ishlab chiqilgan;

yuklangan Korteveg-de Friz va yuklangan modifisirlangan Korteveg-de Friz tenglamalari yechimlarining fazoviy o'zgaruvchilar bo'yicha analitikligi to'g'risidagi natijalar sonli hisoblashlarda qo'llanilishi asoslangan.

Tadqiqot natijalarining ishonchligi matematik fizikaning zamonaviy usullari, matematik tahlil usullaridan usullaridan foydalanilgani hamda matematik mulohazalarning qat'iyiligi bilan asoslangan hamda hisoblash tajribalari natijalari orqali taqqoslangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Dissertatsiyaning ilmiy ahamiyati olingan ilmiy natijalardan chiziqli operatorlarning spektral nazariyasi, gidrodinamika va kvant fizikasida foydalanish mumkinligi bilan izohlanadi.

Dissertatsiyaning amaliy ahamiyati olingan ilmiy natijalardan matematik fizikada noxiziqli evolyutsion tenglamalarni integrallashga tatbiq qilish bilan belgilanadi.

Tadqiqot natijalarining joriy qilinishi. Yuklangan noxiziqli evolyutsion tenglamalarni yechishning to'g'ri va teskari usullari bo'yicha olingan ilmiy natijalar asosida:

Gelfand-Levitan-Marchenko integral tenglamalar sistemasini sonli yechish algoritmlaridan APVV-18-0308-raqamli grant loyihasida moslangan manbali noxiziqli Shredinger tenglamasini sonli yechishda qo'llanilgan (Slovakiyaning Komenskiy nomidagi Universitetning 2022 yil 24 iyundagi ma'lumotnomasi). Ilmiy natijalarning qo'llanilishi chiziqsiz evolyutsion tenglamalarni integrallash imkonini bergan.

Tez kamayuvchi funksiyalar sinfida yuklangan noxiziqli Shredinger tenglamasiga qo'yilgan Koshi masalasi yechimining yagonaligidan MTM2016-75140-P "Oddiy va funksional differensial tenglamalar" grant loyihasida Shredinger tenglamasiga qo'yilgan Koshi masalasini yechishda qo'llanilgan (Ispaniyaning Santiago de Compostela universitetining 2022 yil 27 iyundagi ma'lumotnomasi). Ilmiy natijalarning qo'llanilishi noxiziqli Shredinger tenglamasining sonli-analitik yechimlarini topish imkonini bergan.

Tadqiqot natijalarini aprobatsiyasi. Mazkur tadqiqot natijalari 4 ta xalqaro va 2 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o'tkazilgan.

Tadqiqot natijalarining e'lon qilinganligi. Dissertatsiya mavzusi bo'yicha jami 14 ta ilmiy ish chop etilgan, shulardan, O'zbekiston Respublikasi Oliy attestatsiya komissiyasining falsafa doktori dissertatsiyalarini asosiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 8 ta maqola, jumladan, 4 ta xorijiy va 4 ta respublika jurnallarida nashr etilgan.

Dissertatsiyaning tuzulishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro'yxatidan tashkil topgan. Dissertatsiyaning hajmi 81 betni tashkil etadi.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zaruriyligi asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, mavzu bo‘yicha xorijiy ilmiy tadqiqotlar sharhi, muammoning o‘rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, ob‘ekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma‘lumotlar berilgan.

Dissertatsiyaning **“Dirak operatori uchun sohilish nazariyasi va nochiziqli evolyutsion tenglamalar uchun to‘g‘ri usullar”** deb nomlangan birinchi bobida Dirak operatori uchun teskari spektral masala va evolyutsion tenglamalarni yechishni to‘g‘ri usullari o‘rganilgan.

Butun o‘qda quyidagi tenglamalar sistemasini ko‘rib chiqamiz

$$Lv = \xi v, \quad x \in R, \quad (1)$$

bu yerda $v = (v_1, v_2)^T$ - vektor funksiya va

$$L = i \begin{pmatrix} \frac{d}{dx} & -u(x) \\ -u^*(x) & -\frac{d}{dx} \end{pmatrix}.$$

Bu yerda $u(x)$ potensial quyidagi shartni qanoatlantiradi

$$\int_{-\infty}^{\infty} (1 + |x|) |u(x)| dx < \infty. \quad (2)$$

(1) tenglamalar sistemasining (2) shartni qanoatlantiruvchi quyidagi asimptotikaga ega yechimlari mavjud va bu yechimlar Yost yechimlari deyiladi.

$$\varphi(x, \xi) \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\xi x}, \quad \bar{\varphi}(x, \xi) \sim \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{i\xi x}, \quad x \rightarrow -\infty, \quad (3)$$

$$\psi(x, \xi) \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-i\xi x}, \quad \bar{\psi}(x, \xi) \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{i\xi x}, \quad x \rightarrow \infty. \quad (4)$$

($\bar{\varphi}$ ($\bar{\psi}$) funksiya φ (ψ) funksiyaning kompleks qo‘shmasi). Demak haqiqiy $\xi \in (-\infty, \infty)$ larda $\{\varphi, \bar{\varphi}\}$ va $\{\psi, \bar{\psi}\}$ vektor funksiyalar juftligidan tashkil topgan Vronskiy determinant noldan farqliligidan bu vektor funksiyalar juftligi (1) tenglamalar sistemasining chiziqli erkli yechimlar juftligi bo‘ladi. Shunga ko‘ra, ushbu

$$\begin{cases} \phi = a(\xi)\bar{\psi} + b(\xi)\psi, \\ \bar{\phi} = -\bar{a}(\xi)\psi + \bar{b}(\xi)\bar{\psi}, \end{cases} \quad (5)$$

munosabat o‘rinli bo‘ladi. Bu yerda

$$a(\xi) = W\{\varphi, \psi\} \equiv \varphi_1\psi_2 - \varphi_2\psi_1. \quad (6)$$

Haqiqiy ξ lar uchun quyidagi tenglik o‘rinli

$$a(\xi)\bar{a}(\xi) + b(\xi)\bar{b}(\xi) = 1. \quad (7)$$

$a(\xi)$ va $b(\xi)$ funksiyalar $\text{Im}\xi = 0$ da uzluksiz funksiyalar va $|\xi| \rightarrow \infty$ da

$a(\xi) = 1 + O\left(\frac{1}{|\xi|}\right)$ va $b(\xi) = O\left(\frac{1}{|\xi|}\right)$ asimtotikaga ega. Bundan tashqari $a(\xi)$

funksiya $\text{Im}\xi > 0$ yuqori yarim tekislikka analitik davom qiladi. Yuqori yarim tekislikda cheklita ξ_k ($k = 1, 2, \dots, N$) nollarga ega va L operatorning xos qiymati bo‘ladi. Shuni aytib o‘tishimiz kerakki

$$h_n(x) = \frac{\frac{d}{d\xi}(\varphi - C_n\psi)\Big|_{\xi = \xi_n}}{\dot{a}(\xi_n)}, \quad (8)$$

vektor-funksiya $Lh_n = \xi_n h_n$ tenglamalar sistemasining yechimi bo‘ladi. (6) tenglikni inobatga olsak biz $\text{Im}\xi > 0$ da quyidagi asimtotikaga

$$\begin{aligned} \psi &\sim a(\xi) \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\xi x}, \quad x \rightarrow -\infty, \\ \varphi &\sim a(\xi) \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\xi x}, \quad x \rightarrow \infty, \end{aligned}$$

ega bo‘lamiz. Ushbu asimptotik formulalardan foydalanib, $h_n(x)$ yechimlar uchun asimptotik formulalarni olishimiz mumkin

$$h_n \sim -C_n \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\xi_n x}, \quad x \rightarrow -\infty, \quad h_n \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\xi_n x}, \quad x \rightarrow \infty. \quad (9)$$

Shunga ko‘ra

$$W\{\varphi_n, h_n\} \equiv \varphi_{n1}h_{n2} - \varphi_{n2}h_{n1} = -C_n, \quad (10)$$

bu yerda $\varphi_n = \varphi(x, \xi_n)$.

1-tarif. $\{r(\xi) \equiv \frac{b(\xi)}{a(\xi)}, \xi_k, C_k\}$, $\xi \in R^1$, $\text{Im}\xi_k > 0$, $k = \overline{1, N}$ to‘plamga (1) tenglamalar sistemasi sochilish nazariyasining berilganlari deyiladi.

2-tarif. (1) tenglamalar sistemasi uchun sochilish nazariyasining to‘g‘ri masalasi deb $u(x)$ potensial berilgan holda sochilish nazariyasining berilganlarni topish masalasiga aytiladi.

3-tarif. Sochilish nazariyasini teskari masalasi deb sochilish nazariyasi berilganlari yordamida (1) tenglamalar sistemasining $u(x)$ potensialini aniqlash masalasiga aytiladi.

ψ va $\bar{\psi}$ Yost yechimlari uchun ixtiyoriy $\xi \in R$ larda quyidagi

$$\psi = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\xi x} + \int_x^\infty K(x, s) e^{i\xi s} ds, \quad (11)$$

$$\bar{\Psi} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\xi x} + \int_x^\infty \bar{K}(x, s) e^{-i\xi s} ds, \quad (12)$$

integral tasvir o‘rinli, bunda $K(x, s) = \begin{pmatrix} K_1(x, s) \\ K_2(x, s) \end{pmatrix}$.

Bu yerda yadrolar potensial bilan quyidagi tarzda bog‘langan

$$u(x) = -2K_1(x, x). \quad (13)$$

$K(x, s)$ va $\bar{K}(x, s)$ yadrolar $y > x$ bo‘lganda ushbu

$$\bar{K}(x, y) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} F(x + y) + \int_x^\infty K(x, s) F(s + y) ds = 0,$$

$$K(x, y) - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \bar{F}(x + y) - \int_x^\infty \bar{K}(x, s) \bar{F}(s + y) ds = 0,$$

Gelfand-Levitan-Marchenko integral tenglamalar sistemasini qanoatlantiradi.

Bu yerda

$$F(x) = \frac{1}{2\pi} \int_{-\infty}^\infty \frac{b(\xi)}{a(\xi)} e^{i\xi x} d\xi - i \sum_{j=1}^N C_j e^{i\xi_j x}, \quad \bar{F}(x) = \frac{1}{2\pi} \int_{-\infty}^\infty \frac{\bar{b}(\xi)}{a(\xi)} e^{-i\xi x} d\xi + i \sum_{j=1}^N \bar{C}_j e^{-i\xi_j x}.$$

Birinchi bobning ikkinchi paragrafida noxiziqli evolyutsion tenglamalarni yechishning to‘g‘ri usullari yani Hirota to‘g‘ri usuli va (G'/G) - usuli haqida zaruriy ma‘lumotlar keltirilgan.

Quyidagi

$$F[u] = F(u, u_x, u_t, \dots) = 0$$

noxiziqli xususiy xosilali differensial tenglama berilgan bo‘lsin.

$$u = T[f(x, t, \dots), g(x, t, \dots)],$$

Quyidagi almashtirish yordamida xususiy xosilali differensial tenglamani Hirota D - operator yordamida bichiziqli forma ko‘rinishida yozib olamiz. Takitlash lozimkiy, Kortevag-de Friz tenglamasi kabi ba‘zi xususiy xosilali differensial tenglamalar bitta bichiziqli tenglama ko‘rinishida, modifisirlangan Kortevag-de Friz, noxiziqli Schredinger va boshqa shunga o‘xshash tenglamalarni bichiziqli tenglamalar sistemasi ko‘rinishida yozish mumkin.

4-tarif: $S : C^n \rightarrow C$ differensiullanuvchi funksiyalar fazosi bo‘lsin. $D : S \times S \rightarrow S$ Hirota operatori quyidagi ko‘rinishda

$$[D_x^{m_1} D_t^{m_2} \dots] \{f \bullet g\} = [(\partial_x - \partial_{x'})^{m_1} (\partial_t - \partial_{t'})^{m_2} \dots] f(x, t, \dots) \times g(x', t', \dots) \Big|_{x'=x, t'=t, \dots}, \quad (14)$$

aniqlangan, bu yerda $m_j, j = 1, 2, \dots$ musbat butun son.

D - operatorining qandaydir kombinatsiyasidan foydalanib, biz $F[u]$ tenglamaning bichiziqli formasini D - operatorining ko‘phadi sifatida yozishga harakat qilamiz. Biz bu ko‘phadni $P(D)$ bilan belgilaymiz.

5-tarif: Noxiziqli xususiy xosilali differensial tenglamalar quyidagi ko‘rinishda ifodalansa

$$\sum_{\alpha, \beta=1}^m P_{\alpha\beta}^{\eta}(D) f^{\alpha} f^{\beta} = 0, \quad \eta = 1, \dots, r \quad (15)$$

u holda bu tenglamalar m, r sonlari uchun Hirota bichizikli forma ko‘rinishda ifodalangan deyiladi.

1-tasdiq. $P(D)$ operatorga differensiallanuvchi f va g funksiyalar ta’sir qilsin. U holda quyidagi tenglik o‘rinli

$$P(D)\{f \cdot g\} = P(-D)\{f \cdot g\}. \quad (16)$$

2-tasdiq. $P(D)$ operatorga differensiallanuvchi f va $g = 1$ funksiyalar ta’sir qilsin. U holda quyidagi tenglik o‘rinli

$$P(D)\{f \cdot 1\} = P(\partial) f, \quad P(D)\{1 \cdot f\} = P(-\partial) f. \quad (17)$$

3-tasdiq. $P(D)$ operatorga eksponensial funksiyalar e^{θ_1} va e^{θ_2} ta’sir qilsin, bu yerda $\theta_j = k_j x + \dots r_j z + l_j y + \alpha_j$ va $k_j, \dots, r_j, l_j, \alpha_j$ ($j = 1, 2, \dots$). U holda quyidagi tenglik o‘rinli

$$P(D)\{e^{\theta_1} \cdot e^{\theta_2}\} = P(k_1 - k_2, \dots, r_1 - r_2, l_1 - l_2) e^{\theta_1 + \theta_2}. \quad (18)$$

1-natija. Noldan farqli a konstanta uchun $P(D)\{a \cdot a\} = 0$ o‘rinli bo‘lsa, u holda 3-tasdiqga asosan $P(0, 0, \dots, 0) = 0$ ega bo‘lamiz.

Izoh. $P(D)\{f \cdot f\}$ uchun quyidagi oddiy misollarni ko‘rib chiqmiz

$$D_x\{f \cdot f\} = f_x f - f f_x = 0,$$

$$D_t D_x^2\{f \cdot f\} = f_{xxt} f - f_{xx} f_t - f_{xt} f_x + f_x f_{xt} - f_{xt} f_x + f_x f_{xt} + f_t f_{xx} - f f_{xxt} = 0.$$

Ushbu paragrafda (G' / G) - usulining asosiy qadamlari keltirib o‘tilgan.

Bizga quyidagi nochizikli xususiy xosilali differensial tenglama berilgan bo‘lsin

$$F(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \dots) = 0, \quad (19)$$

bu yerda $u = u(x, t)$ noma’lum funksiya, F - u va uning xususiy xosilalariga bog‘liq bo‘lgan nochizikli funksiya.

1-qadam. $u(x, t)$ funksiya uchun quyidagi

$$u(x, t) = u(\xi), \quad \xi = kx + \Omega(t), \quad (20)$$

almashtirishdan foydalanamiz, bu yerga k - parametr, $\Omega(t)$ - uzluksiz funksiya.

Yechimning (20) almashtirishini inobatga olsak (19) tenglamani

$$P(u, u', u'', u''', \dots) = 0, \quad (21)$$

oddiy differensial tenglamaga keltirish imkonini beradi, bu yerda $u' = \frac{du(\xi)}{d\xi}$,

$$u'' = \frac{d^2 u(\xi)}{d\xi^2}, \quad u''' = \frac{d^3 u(\xi)}{d\xi^3}, \quad \dots$$

2-qadam. Faraz qilaylik, (21) tenglamaning yechimini $\left(\frac{G'}{G}\right)$ ko‘phad

ko‘rinishida ifodalash mumkin:

$$u(\xi) = \sum_{j=0}^m a_j \left(\frac{G'}{G} \right)^j, \quad (22)$$

bu yerda G' funksiya $G(\xi)$ funksiyaning ξ bo'yicha xosilasi, a_j ($j=0,1,2,\dots,m$) - keyin aniqlanadigan konstantalar, m - musbat butun son, $G = G(\xi)$ funksiya quyidagi oddiy differensial tenglamaning yechimi

$$G'' + \lambda G' + \mu G = 0, \quad (23)$$

bu yerda λ va μ ixtiyoriy konstantalar.

3-qadam. (20) funksiyaning (21) tenglamaga qo'yib, ikkinchi tartibli chiziqli oddiy differensial tenglama (23) yordamida va (G'/G) bir xil tartibli hadlarni yig'ish orqali (G'/G) nisbatan ko'phad ko'rinishida tenglamaga ega bo'lamiz. (G'/G) ning bir xil darajasi oldidagi koeffitsientlarini nolga tenglashtirish orqali $\Omega(t)$, a_j , $j=1,2,\dots,m$ larda nisbatan tenglamalar sistemasiga ega bo'lamiz.

4-qadam. $\Omega(t)$, a_j , $j=1,2,\dots,m$ larni topib, (22), (23) tengliklarni inobatga olib (19) xususiy xosilali differensial tenglamaning aniq yechimini topamiz.

Dissertatsiyaning "**Teskari masala usuli yordamida yuklangan nochiziqli Shredinger tenglamasini integrallash**" deb nomlangan ikkinchi bobida Dirak operatori yordamida tez kamayuvchi funksiyalar sinfida yuklangan nochiziqli Shredinger tenglamasini to'la integrallanuvchiligi isbotlangan va sochilish nazariyasining asosiy integral tenglamasi Gelfand-Levitan-Marchenko integral tenglamasining sonli yechimlari topilgan.

Ikkinchi bobning birinchi paragrafida yuklangan nochiziqli Shredinger tenglamasi "tez kamayuvchi" funksiyalar sinfida integrallangan.

Ushbu

$$iu_t + 2|u|^2 u + u_{xx} - i\gamma(t)(u(0,t) + u^*(0,t))u_x = 0, \quad (24)$$

yuklangan nochiziqli Shredinger tenglamasi uchun qo'yilgan

$$u(x,0) = u_0(x), \quad (25)$$

Koshi masalasini ko'rib chiqamiz. Bu yerda $\gamma(t)$ - nolga aylanmaydigan ixtiyoriy funksiya, $u^*(0,t)$ funksiya $u(0,t)$ funksiyaning kompleks qo'shmasi.

Boshlang'ich shartdagi $u_0(x)$ funksiya quyidagi xossalarga ega bo'lsin:

$$1. \int_{-\infty}^{\infty} (1+|x|)|u_0(x)|dx < \infty;$$

$$2. L(0)y \equiv i \begin{pmatrix} \frac{d}{dx} & -u_0(x) \\ -u_0^*(x) & -\frac{d}{dx} \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \xi \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \quad (26)$$

tenglama N ta oddiy xos qiymatga ega bo'lsin.

Bu paragrafda (24)-(26) masalaning ushbu $u(x,t)$ yechimi quyidagi

$$\int_{-\infty}^{\infty} \left((1+|x|)|u_0(x)| + \sum_{j=1}^3 \left| \frac{\partial^j u(x,t)}{\partial x^j} \right| \right) dx < \infty, \quad t \geq 0$$

funksiyalar sinfida mavjud deb hisoblaniladi va bu yechimni topish usulini keltirib chiqarish talab qilinadi. Shu maqsadda $L(t)$ operator uchun sochilish nazariyasi berilganlarining vaqt bo'yicha evolyutsiyalari aniqlanadi. Olingan asosiy natija quyidagi teoremlarda ifodalangan.

1-teorema. Agar $u(x,t)$ funksiya (24)-(26) masalaning yechimi bo'lsa, u holda u yagona.

2-teorema. Agar $u(x,t)$ funksiya (24)-(26) masalaning yechimi bo'lsa, u holda $L(t)$ operatorning sochilish nazariyasi berilganlari uchun quyidagi munosabatlar o'rinli:

$$\frac{dr^+}{dt} = \left(4i\xi^2 - 2\xi\gamma(t)(u(0,t) + u^*(0,t)) \right) r^+, \quad \text{Im } \xi = 0,$$

$$\frac{dC_n}{dt} = \left(4i\xi_n^2 + 2i\xi_n\gamma(t)(u(0,t) + u^*(0,t)) \right) C_n,$$

$$\frac{d\xi_n}{dt} = 0, \quad n = 1, 2, 3, \dots, N.$$

Olingan munosabatlar $L(t)$ operatorning sochilish nazariyasi berilganlarini to'liq ifodalaydi va ular yordamida (24)-(26) masalaning yechimi teskari masala usuli yordamida topiladi.

Quyida keltirilgan algoritm yordamida yechimni topishimiz mumkin. Bizga yuqoridagi masalaning shartlarini qanoatlantiruvchi $u_0(x)$ berilgan bo'lsin.

1. Potensialni $u_0(x)$ bo'lgan $L(0)$ operator uchun to'g'ri masalani yechib, $\{r(\xi, 0), \xi_1(0), \xi_2(0), \dots, \xi_N(0), C_1(0), C_2(0), \dots, C_N(0)\}$ sochilish nazariyasining berilganlarini topib olamiz.
2. 2-teoremaga ko'ra, sochilish nazariyasining berilganlarini vaqt bo'yicha evolyutsiyasini $\{r(\xi, t), \xi_1(t), \xi_2(t), \dots, \xi_N(t), C_1(t), C_2(t), \dots, C_N(t)\}$, $t > 0$ aniqlaymiz.
3. Gelfand-Levitan-Marchenko integral tenglamasiga asoslangan teskari masala usulidan foydalanib, $u(x,t)$ potensialni $\{r(\xi, t), \xi_1(t), \xi_2(t), \dots, \xi_N(t), C_1(t), C_2(t), \dots, C_N(t)\}$ yordamida tiklaymiz.

Ikkinchi bobning ikkinchi paragrafida sochilish nazariyasining asosiy integral tenglamasi Gelfand-Levitan-Marchenko integral tenglamasining sonli yechimlarini topish ko'rsatilgan.

Ushbu

$$\begin{cases} \bar{K}_2(x, y) + \int_0^\infty K_1(x, s+x) F(s+x+y) ds = 0, \\ -K_1(x, y) + \bar{F}(x+y) + \int_0^\infty \bar{K}_2(x, s+x) \bar{F}(s+x+y) ds = 0, \end{cases} \quad (27)$$

Gelfand-Levitan-Marchenko integral tenglamasining Nystrom usulidan foydalanib sonli yechimlarini topishni ko'rib chiqamiz, bu yerda

$$F(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\xi) e^{i\xi x} d\xi - i \sum_{k=1}^N c_k e^{i\xi_k x}. \quad (28)$$

(27) integral tenglamalar sistemasining yadrosi $s + x$ and $s + x + y$ larga bog‘liq, shuning uchun tugun nuqtalarni quyidagi ko‘rinishda yozib olamiz

$$\begin{aligned} s_j &= (j-1)\tau, \quad \beta_{k+j} = x_k + s_j, \quad \alpha_{k+j+p} = \beta_{k+j} + \beta_{p+k}, \\ \alpha_{k+p} &= x_k + \beta_{p+k}, \quad k = 1, 2, \dots, N_x, \quad j = 1, 2, \dots, N_y. \end{aligned} \quad (29)$$

Simpson kvadraturasi formulasini va (29) to‘r nuqtalarni inobatga olib biz (27) integral tenglamalar sistemasini algebraik tenglamalar sistemasini ko‘rinishida yozib olamiz:

$$\begin{cases} K_2^*(x_k, \beta_{k+p}) + \sum_{j=1}^{N_y} d_j K_1(x_k, \beta_{k+j}) F(\alpha_{k+j+p}) = 0, \\ \sum_{j=1}^{N_y} d_j K_2^*(x_k, \beta_{k+j}) F^*(\alpha_{k+j+p}) - K_1(x_k, \beta_{k+p}) = -F^*(\alpha_{k+p}), \end{cases} \quad (30)$$

bu yerda $\{d_i\}_{i=0}^{N_y} = \frac{\tau}{3} \{1, 4, 2, 4, 2, 4, \dots, 1\}$, τ - y bo‘yicha qadami.

(30) tenglamalar sistemasini matritsa ko‘rinishida quyidagicha yozishimiz mumkin

$$\begin{bmatrix} I_{N_y} & HD \\ H^*D & -I_{N_y} \end{bmatrix} \begin{bmatrix} k_2 \\ k_1 \end{bmatrix} = \begin{bmatrix} 0 \\ -f_{N_y} \end{bmatrix}, \quad (31)$$

bu yerda $D = \text{diag}\{d_j\}_{j=1}^{N_y}$, $I_{N_y} = \text{diag}\{1\}_{j=1}^{N_y}$,

$$H = (F(\alpha_{k+j+p}))_{p,j=1}^{N_y}, \quad f_{N_y} = (F(\alpha_{k+p}))_{p=1}^{N_y}, \quad (32)$$

va noma‘lum vektor funksiyalar

$$k_2 = (K_2(x_k, \beta_{k+p}))_{p=1}^{N_y}, \quad k_1 = (K_1(x_k, \beta_{k+p}))_{p=1}^{N_y}. \quad (33)$$

(31) algebraik tenglamalar sistemasini quyidagi shaklda ifodalash mumkin

$$\begin{cases} (DH^*DHD + D)k_1 = Df_{N_y}, \\ k_2 = -HDk_1, \end{cases}$$

bu yerda $DH^*DHD + D$ matritsa simmetrik shuning uchun tenglamalar sistemasini gradient usuli orqali yechish mumkin. H matritsa $N_y \times N_y$ o‘lchamli Hankel matritsasi, $(H)_{ij} = F(\alpha_{k+i+j})$. (13) va (33) inobatga olib $u(x)$ potensialni k_1 orqali ifodalash mumkin.

Dissertasiyaning “**Nochiziqli evolyutsion tenglamalarni yechishning to‘g‘ri usullari**” deb nomlangan uchinchi bobida moslangan manbali nochiziqli Shredinger tenglamasi va yuklangan nochiziqli Shredinger tenglamasining tez kamayuvchi funksiyalar sinfida soliton yechimlarini, yuklangan Kortevog-de Friz

va yuklangan modifisirlangan Kortevog-de Friz tenglamasining aniq yechimlarini to'g'ri usullar yordamida topish algoritmi keltirilgan.

Uchinchi bobning birinchi paragrafida moslangan manbali nochiziqli Shredinger tenglamasining tez kamayuvchi funksiyalar sinfida Hirota usuli yordamida soliton yechimlari topilgan.

Quyidagi

$$iu_t + 2|u|^2 u + u_{xx} = 2i \sum_{j=1}^N (\varphi_{1j}^2 - \bar{\varphi}_{2j}^2), \quad (34)$$

$$\begin{cases} \varphi_{1j,x} = i\xi_j \varphi_{1j} - u \varphi_{2j}, \\ \varphi_{2j,x} = -i\xi_j \varphi_{2j} + \bar{u} \varphi_{1j}, \quad j = 1, 2, \dots, N, \end{cases} \quad (35)$$

moslangan manbali nochiziqli Shredinger tenglamasini qaraymiz, bu yerda ξ_j $j = 1, 2, \dots, N$ xos qiymatlar.

Bu paragrafda (34)-(35) masalaning ushbu $u(x, t)$ yechimi quyidagi

$$\int_{-\infty}^{\infty} \left((1 + |x|) |u(x, t)| + \sum_{j=1}^2 \left| \frac{\partial^j u(x, t)}{\partial x^j} \right| \right) dx < \infty, t \geq 0 \quad (36)$$

funksiyalar sinfida mavjud deb hisoblaniladi va bu yechimni topish usulini keltirib chiqarish talab qilinadi. (35) sistemaning ξ_j $j = 1, 2, \dots, N$ xos qiymatlarga mos keluvchi $\Phi_j = (\varphi_{1j}, \varphi_{2j})^T$ xos vektorlari uchun quyidagi normallovchi shart o'rinli bo'lsin

$$\int_{-\infty}^{\infty} \varphi_{1j} \varphi_{2j} dx = \beta_j^2(t), \quad j = 1, 2, \dots, N. \quad (36')$$

bu yerda $\beta_j(t)$, $j = 1, 2, \dots, N$ oldindan berilgan uzluksiz funksiya.

Moslangan manbali nochiziqli Shredinger tenglamasining soliton yechimlarini Hirota usuli yordamida topishni ko'rib chiqamiz. Ushbu almashtirishlar

$$u = \frac{g}{f}, \quad \varphi_{1j} = \frac{p_j}{f}, \quad \varphi_{2j} = \frac{h_j}{f}, \quad j = 1, 2, \dots, N, \quad (37)$$

yordamida (34)-(35) masalani quyidagi bichiziqli forma ko'rinishida yozib olamiz

$$(iD_t + D_x^2) g \cdot f = 2i \sum_{j=1}^N (p_j^2 - \bar{h}_j^2), \quad (38)$$

$$D_x^2 f \cdot f = 2g \cdot \bar{g}, \quad (39)$$

$$\begin{cases} D_x p_j \cdot f = -i\xi_j p_j f + g h_j, \\ D_x h_j \cdot f = i\xi_j h_j f - \bar{g} p_j, \end{cases} \quad (40)$$

bu yerda \bar{g} va \bar{h} funksiyalar g va h funksiyalarning kompleks qo'shmasi, Hirota bichiziqli operatori quyidagi ko'rinishda aniqlangan

$$D_x^m D_t^n g(x,t) \cdot f(x,t) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'} \right)^n g(x,t) f(x',t') \Big|_{x=x', t=t'}. \quad (41)$$

(38)-(40) tenglamalarni yechishda f , g , p_j va h_j funksiyalarni quyidagi ko‘rinishda izlaymiz:

$$f = 1 + \chi^2 f^{(1)} + \chi^4 f^{(2)} + \dots, \quad (42)$$

$$g = \chi g^{(1)} + \chi^3 g^{(2)} + \dots, \quad (43)$$

$$p_j = \chi p_j^{(1)} + \chi^3 p_j^{(2)} + \dots, \quad (44)$$

$$h_j = h_j^{(1)}, \quad (45)$$

bu yerda χ parametr. (42)-(45) funksiyalarni (38)-(40) tenglamalarga qo‘yish va χ ning bir xil darajali koeffitsientlarini nolga tenglashtirish orqali $f^{(k)}$, $g^{(k)}$, $p_j^{(k)}$ va $h_j^{(k)}$, ($k=1,2,\dots$) larga nisbatan tenglamalar sistemasiga ega bo‘lamiz. Ularni yechish va (37) almashtirishni inobatga olsak (34)-(35) masalaning yechimiga ega bo‘lamiz.

Moslangan manbali nochiziqli Shredinger tenglamasining bir soliton yechimini topish uchun f va g funksiyalarni quyidagi ko‘rinishda izlaymiz

$$g = \chi g^{(1)}, \quad f = 1 + \chi^2 f^{(1)}.$$

Yuqoridagi f va g funksiyalarni (39) tenglamaga qo‘llash va χ ning bir xil darajalarining koeffitsientlarini tenglashtirib, $f^{(1)}$ va $g^{(1)}$ ga nisbatan tenglamalar sistemasini yozib olamiz va ularni yechish orqali quyidagiga ega bo‘lamiz

$$g^{(1)} = e^{\eta_1}, \quad f^{(1)} = e^{\eta_1 + \bar{\eta}_1 + a_{11}}, \quad (46)$$

bu yerda $\eta_1 = k_1 x + \gamma_1(t)$, $a_{11} = \ln \frac{1}{(k_1 + \bar{k}_1)^2}$, k_1 - konstanta va $\gamma_1(t)$ uzluksiz funksiya. Keyin qadam p_1 va h_1 funksiyalarni topishdan iborat bo‘ladi. Buning uchun p_1 va h_1 funksiyalar

$$p_1 = \chi p_j^{(1)}, \quad h_1 = h_j^{(1)}$$

ko‘rinishda bo‘ladi. Yuqordagi p_1 va h_1 funksiyalarni (40) tenglamaga qo‘llash va χ ning bir xil darajalarining koeffitsientlarini tenglashtirib, p_1 va h_1 ga nisbatan tenglamalar sistemasini yozib olamiz va ularni yechish orqali quyidagiga ega bo‘lamiz

$$\begin{aligned} p_1^{(1)} &= e^{(k_1 + \bar{k}_1 - i\xi_1)x + \Omega_1(t)}, \\ h_1^{(1)} &= (k_1 + \bar{k}_1) e^{(\bar{k}_1 - i\xi_1)x + \Omega_1(t) - \gamma_1(t)}, \end{aligned} \quad (47)$$

bu yerda $\Omega_1(t)$ uzluksiz funksiya.

(46) va (47) tengliklarni inobatga olsak biz quyidagilarga ega bo‘lamiz

$$\begin{aligned}
f &= 1 + e^{\eta_1 + \eta_2 + a_{11}}, \\
g &= e^{\eta_1}, \\
p_1 &= e^{(k_1 + \bar{k}_1 - i\xi_1)x + \Omega_1(t)}, \\
h_1 &= (k_1 + \bar{k}_1)e^{(\bar{k}_1 - i\xi_1)x + \Omega_1(t) - \gamma_1(t)}.
\end{aligned} \tag{48}$$

(48) funksiyalar va (38) tenglamadan foydalanib $\Omega_1(t)$, $\gamma_1(t)$ funksiyalarni va k_1 konstantani topish mumkin

$$\begin{aligned}
\Omega_1(t) &= \ln \beta_1(t) + \frac{1}{2}(\bar{\gamma}_1(t) + a_{11}) + \gamma_1(t), \\
\gamma_1(t) &= -4i\bar{\xi}_1^2 t - 2 \int_0^t \bar{\beta}_1^2(\tau) d\tau + \gamma_1(0), \\
k_1 &= -2i\bar{\xi}_1.
\end{aligned} \tag{49}$$

(37), (48) va (49) tengliklarni inobatga olib biz (34)-(35) masalaning bir soliton yechimini yozib olamiz

$$\begin{aligned}
u(x,t) &= \frac{e^{-2i\bar{\xi}_1 x + \gamma_1(t)}}{1 + e^{(-2i\bar{\xi}_1 + 2i\xi_1)x + \gamma_1(t) + \bar{\gamma}_1(t) + a_{11}}}, \\
\varphi_{11}(x,t) &= \beta_1(t) \frac{e^{(-2i\bar{\xi}_1 + i\xi_1)x + \gamma_1(t) + \bar{\gamma}_1(t) + a_{11}}}{1 + e^{(-2i\bar{\xi}_1 + 2i\xi_1)x + \gamma_1(t) + \bar{\gamma}_1(t) + a_{11}}}, \\
\varphi_{21}(x,t) &= \beta_1(t) \frac{(-2i\bar{\xi}_1 + 2i\xi_1)e^{i\xi_1 x + \bar{\gamma}_1(t) + a_{11}}}{1 + e^{(-2i\bar{\xi}_1 + 2i\xi_1)x + \gamma_1(t) + \bar{\gamma}_1(t) + a_{11}}}.
\end{aligned} \tag{50}$$

Moslangan manbali nochiziqli Shredinger tenglamasining ikki soliton yechimini topish uchun f , g , p_j va h_j ($j=1,2$) funksiyalarni quyidagi ko'rinishda izlaymiz

$$\begin{aligned}
g &= \chi g^{(1)} + \chi^3 g^{(2)}, \quad f = 1 + \chi^2 f^{(1)} + \chi^4 f^{(2)}. \\
p_j &= \chi p_j^{(1)} + \chi^3 p_j^{(2)}, \quad h_j = h_j^{(1)}.
\end{aligned}$$

Yuqordagi jarayonni qo'llash orqali biz moslangan manbali nochiziqli Shredinger tenglamasining ikki soliton yechimini quyidagi ko'rinishda yozishimiz mumkin

$$u(x,t) = \frac{g^{(1)} + g^{(2)}}{1 + f^{(1)} + f^{(2)}}, \tag{51}$$

$$\varphi_{1j} = \frac{p^{(1)} + p^{(2)}}{1 + f^{(1)} + f^{(2)}}, \quad \varphi_{2j} = \frac{h^{(1)}}{1 + f^{(1)} + f^{(2)}}, \quad j = 1, 2.$$

Bunda

$$f^{(1)} = e^{\eta_1 + \bar{\eta}_1 + a_{11}} + e^{\eta_1 + \bar{\eta}_2 + a_{12}} + e^{\eta_2 + \bar{\eta}_1 + a_{21}} + e^{\eta_2 + \bar{\eta}_2 + a_{22}}, \quad g^{(1)} = e^{\eta_1} + e^{\eta_2}, \tag{52}$$

$$f^{(2)} = e^{\eta_1 + \bar{\eta}_1 + \eta_2 + \bar{\eta}_2 + r}, \quad g^{(2)} = e^{\eta_1 + \bar{\eta}_1 + \bar{\eta}_2 + \delta_1} + e^{\eta_1 + \bar{\eta}_2 + \eta_2 + \delta_2}, \tag{53}$$

$$\begin{aligned}
p_j^{(1)} &= \exp((k_1 + \bar{k}_1 - i\xi_j)x - \gamma_2(t) - \bar{\gamma}_2(t) + \gamma_1(t) + \bar{\gamma}_1(t) + \Omega_j(t) - r + a_{11}) + \\
&+ \exp((k_1 + \bar{k}_2 - i\xi_j)x - \gamma_1(t) - \bar{\gamma}_1(t) + \gamma_2(t) + \bar{\gamma}_2(t) + \Omega_j(t) - r + a_{12}) +
\end{aligned}$$

$$\begin{aligned}
& + \exp((k_2 + \bar{k}_1 - i\xi_j)x - \gamma_1(t) - \bar{\gamma}_2(t) + \bar{\gamma}_1(t) + \gamma_2(t) + \Omega_j(t) - r + a_{21}) + \\
& + \exp((k_2 + \bar{k}_2 - i\xi_j)x - \gamma_1(t) - \bar{\gamma}_1(t) + \gamma_2(t) + \bar{\gamma}_2(t) + \Omega_j(t) - r + a_{22}), \quad (54)
\end{aligned}$$

$$p_j^{(2)} = \exp((k_1 + \bar{k}_1 + k_2 + \bar{k}_2 - i\xi_j)x + \Omega_j(t)), \quad (55)$$

$$\begin{aligned}
h_j^{(1)} = & \frac{\exp((k_1 + \bar{k}_1 - i\xi_j)x - \gamma_2(t) - \bar{\gamma}_2(t) + \gamma_1(t) + \bar{\gamma}_1(t) + \Omega_j(t) - r)}{(k_1 + \bar{k}_1)(e^{\eta_1} + e^{\eta_2})} + \\
& + \frac{\exp((k_1 + \bar{k}_2 - i\xi_j)x - \gamma_2(t) - \bar{\gamma}_2(t) + \gamma_1(t) + \bar{\gamma}_1(t) + \Omega_j(t) - r)}{(k_1 + \bar{k}_2)(e^{\eta_1} + e^{\eta_2})} + \\
& + \frac{\exp((k_2 + \bar{k}_1 - i\xi_j)x - \gamma_2(t) - \bar{\gamma}_2(t) + \gamma_1(t) + \bar{\gamma}_1(t) + \Omega_j(t) - r)}{(k_2 + \bar{k}_1)(e^{\eta_1} + e^{\eta_2})} + \\
& + \frac{\exp((k_2 + \bar{k}_2 - i\xi_j)x - \gamma_2(t) - \bar{\gamma}_2(t) + \gamma_1(t) + \bar{\gamma}_1(t) + \Omega_j(t) - r)}{(k_2 + \bar{k}_2)(e^{\eta_1} + e^{\eta_2})}, \quad (j=1,2), \quad (56)
\end{aligned}$$

bu yerda $\eta_j = k_j x + \gamma_j(t)$, ($j=1,2$), k_j konstantalar, $\gamma_j(t)$, $\Omega_j(t)$ ($j=1,2$) uzluksiz funksiyalar va

$$a_{mn} = \ln \frac{1}{(k_m + \bar{k}_n)^2}, \quad m, n = 1, 2$$

$$\delta_j = \ln \left(\frac{B_j}{q_j} \right), \quad j = 1, 2,$$

$$q_1 = (k_1 + \bar{k}_1)(k_2 + \bar{k}_1), \quad q_2 = (k_1 + \bar{k}_2)(k_2 + \bar{k}_2),$$

$$B_1 = \frac{(k_1 + \bar{k}_1 + k_2 + \bar{k}_2)}{(k_1 + \bar{k}_1)^2 (k_2 + \bar{k}_1)^2}, \quad B_2 = \frac{(k_1 + \bar{k}_1 + k_2 + \bar{k}_2)}{(k_1 + \bar{k}_2)^2 (k_2 + \bar{k}_2)^2},$$

$$r = \frac{(k_2 - k_1)^2 (\bar{k}_2 - \bar{k}_1)^2}{(k_1 + \bar{k}_1)^2 (k_1 + \bar{k}_2)^2 (k_2 + \bar{k}_1)^2}.$$

(52)-(56) tengliklarni (38) tenglamaga qo'llash orqali $\gamma_j(t)$, $\Omega_j(t)$ ($j=1,2$) funksiyalarni va k_1 , k_2 konstantalarni topish mumkin

$$\Omega_j(t) = \ln \beta_j(t) + \frac{1}{2} (\bar{\gamma}_j(t) - r + \delta_j) + \gamma_1(t) + \gamma_2(t), \quad (j=1,2), \quad (57)$$

$$\gamma_j(t) = -4i\bar{\xi}_j^2 t - 2 \int_0^t \bar{\beta}_j^2(\tau) d\tau + \gamma_j(0), \quad (j=1,2), \quad (58)$$

$$k_j = -2i\bar{\xi}_j, \quad (j=1,2). \quad (59)$$

Olingan asosiy natija quyidagi teoremda ifodalangan.

3-teorema. Agar $u(x,t)$ yechim (36) shartni va $\varphi_{1j}(x,t)$, $\varphi_{2j}(x,t)$ funksiyalar uchun (36) shart o'rinli bo'lsa u holda (34)-(35) masalaning bir solitonli yechimi (50), ikki solitonli yechimi (51) ko'rinishda bo'ladi.

Uchinchi bobning ikkinchi paragrafida yuklangan noxiziqli Shredinger tenglamasining soliton yechimlari Hirota to'g'ri usulida topilgan.

Quyidagi

$$iu_t + 2|u|^2 u + u_{xx} + h(t)u_x = 0, \quad x \in R, \quad t \geq 0, \quad (60)$$

yuklangan noxiziqli Shredinger tenglamasini qaraymiz, bu yerda $h(t) = -\gamma(t)u(0,t)$ va $u(x,t)$ noma'lum funksiya, $\gamma(t)$ - berilgan uzluksiz funksiya.

Bu paragrafda (60) tenglamaning $u(x,t)$ yechimini quyidagi

$$\int_{-\infty}^{\infty} \left((1+|x|)|u(x,t)| + \sum_{j=1}^2 \left| \frac{\partial^j u(x,t)}{\partial x^j} \right| \right) dx < \infty, \quad t \geq 0, \quad (61)$$

funksiyalar sinfida mavjud deb hisoblaniladi va bu yechimni topish usulini keltirib chiqarish talab qilinadi.

Yuklangan noxiziqli Shredinger tenglamasining soliton yechimlarini Hirota usuli yordamida topishni ko'rib chiqamiz. Ushbu almashtirish

$$u = \frac{g}{f}, \quad (62)$$

yordamida (60) tenglamani quyidagi bichiziqli forma ko'rinishida yozib olamiz:

$$\begin{cases} iD_t g \cdot f + D_x^2 g \cdot f + h(t)D_x g \cdot f = 0, \\ D_x^2 f \cdot f = 2\bar{g}g, \end{cases} \quad (63)$$

bu yerda \bar{g} funksiya g funksiyaning kompleks qo'shmasi, Hirota bichiziqli operatori quyidagi ko'rinishda aniqlangan:

$$D_x^m D_t^n g(x,t) \cdot f(x,t) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'} \right)^n g(x,t) f(x',t') \Big|_{x=x', t=t'}.$$

Yuklangan noxiziqli Shredinger tenglamasining bir solitonli yechimini topish uchun f va g funksiyalarni quyidagi ko'rinishda izlaymiz

$$f = 1 + \chi^2 f^{(1)}, \quad g = \chi g^{(1)}. \quad (64)$$

Yuqoridagi f va g funksiyalarni (63) tenglamaga qo'llab va χ ning bir xil darajalarining koeffitsientlarini tenglashtirib, $f^{(1)}$ va $g^{(1)}$ ga nisbatan tenglamalar sistemasini yozib olamiz va ularni yechish orqali quyidagiga ega bo'lamiz

$$g^{(1)} = e^{\xi_1}, \quad f^{(1)} = \frac{1}{(k_1 + \bar{k}_1)^2} e^{\xi_1 + \bar{\xi}_1}, \quad (65)$$

bu yerda $\xi_1 = k_1 x + \Omega_1(t)$, k_1 - ixtiyoriy konstanta va $\Omega_1(t)$ uzluksiz funksiya. Keyingi qadam $\Omega_1(t)$ funksiyani topishdan iborat bo'ladi. (63) va (64) ifodadan foydalanib $\Omega_1(t)$ funksiyani topish mumkin

$$\Omega_1(t) = ik_1^2 t + ik_1 \int_0^t \gamma(\tau) u(0, \tau) d\tau, \quad (66)$$

(64)-(66) va (62) almashtirishni inobatga olib, biz (60) tenglamaning bir soliton yechimini yozib olamiz

$$u(x,t) = \frac{e^{k_1 x + ik_1^2 t + ik_1 \int_0^t \gamma(\tau) u(0,\tau) d\tau}}{1 + \frac{1}{(k_1 + \bar{k}_1)^2} e^{(k_1 + \bar{k}_1)x + i(k_1^2 + \bar{k}_1^2)t + i(k_1 + \bar{k}_1) \int_0^t \gamma(\tau) u(0,\tau) d\tau}}. \quad (67)$$

Moslangan manbali nohiziqli Shredinger tenglamasining ikki soliton yechimini topish uchun f va g funksiyalarni quyidagi ko‘rinishda izlaymiz

$$g = \chi g^{(1)} + \chi^3 g^{(2)}, \quad f = 1 + \chi^2 f^{(1)} + \chi^4 f^{(2)}.$$

Yuqoridagi f va g funksiyalarni (64) tenglamaga qo‘llab va χ ning bir xil darajalarining koeffitsientlarini tenglashtirib, $f^{(1)}$, $f^{(2)}$, $g^{(1)}$ va $g^{(2)}$ ga nisbatan tenglamalar sistemasini yozib olamiz va ularni yechish orqali quyidagilarga ega bo‘lamiz:

$$f^{(1)} = e^{\xi_1 + \bar{\xi}_1 + a_{11}} + e^{\xi_1 + \bar{\xi}_2 + a_{12}} + e^{\xi_2 + \bar{\xi}_1 + a_{21}} + e^{\xi_2 + \bar{\xi}_2 + a_{22}}, \quad g^{(1)} = e^{\xi_1} + e^{\xi_2}, \quad g^{(1)} = e^{\eta_1} + e^{\eta_2}, \quad (68)$$

$$f^{(2)} = e^{\xi_1 + \bar{\xi}_1 + \xi_2 + \bar{\xi}_2 + r}, \quad g^{(2)} = e^{\xi_1 + \bar{\xi}_1 + \xi_2 + \delta_1} + e^{\xi_1 + \xi_2 + \bar{\xi}_2 + \delta_2}, \quad (69)$$

bu yerda $\xi_j = k_j x + \Omega_j(t)$, $\Omega_j(t) = ik_j^2 t + ik_j \int_0^t \gamma(\tau) u(0,\tau) d\tau$, $j=1,2$, k_1 va k_2

konstantalar va

$$a_{mn} = \ln \frac{1}{(k_m + \bar{k}_n)^2}, \quad m, n = 1, 2$$

$$\delta_1 = \ln \left(\frac{1}{(k_1 + \bar{k}_1)^2} + \frac{1}{(k_2 + \bar{k}_1)^2} \right), \quad \delta_2 = \ln \left(\frac{1}{(k_2 + \bar{k}_2)^2} + \frac{1}{(k_1 + \bar{k}_2)^2} \right),$$

$$r = \frac{(k_2 + \bar{k}_2)^2 (k_2 + \bar{k}_1)^2 + (k_1 + \bar{k}_2)^2 (k_2 + \bar{k}_1)^2 + (k_1 + \bar{k}_2)^2 (k_2 + \bar{k}_2)^2}{(k_1 + \bar{k}_2)^2 (k_2 + \bar{k}_2)^2 (k_2 + \bar{k}_1)^2 (k_1 + \bar{k}_1 + k_2 + \bar{k}_2)^2}, \quad j=1,2.$$

(68), (69) funksiyalarni va (62) tengliklarni inobatga olib biz (60) tenglamaning ikki soliton yechimini yozib olamiz

$$u = \frac{g^{(1)} + g^{(2)}}{1 + f^{(1)} + f^{(2)}}. \quad (70)$$

Olingan asosiy natija quyidagi teoremda ifodalangan.

4-teorema. Agar $u(x,t)$ yechim (61) shartni qanoatlantirsa u holda (60) tenglamaning bir solitonli yechimi (67), ikki solitonli yechimi (70) ko‘rinishda bo‘ladi.

Uchinchi bobning uchinchi paragrafida yuklangan Korteveg-de Friz tenglamasining aniq yechimini topish algoritmi ko‘rsatilgan.

Quyidagi

$$u_t - 6uu_x + u_{xxx} - \gamma(t)u(0,t)u_x = 0, \quad t \geq 0, \quad x \in \mathbb{R}, \quad (71)$$

yuklangan Korteveg-de Friz tenglamasi berilgan bo'lsin. Bu yerda $\gamma(t)$ oldindan berilgan uzluksiz funksiya. (G' / G) usulini (71) tenglamaga qo'llash orqali biz uch xil turdagi yechimini olamiz. Bu quyidagi teoramada ifodalangan.

5-teorema. Agar $(\lambda^2 - 4\mu) > 0$ bo'lsa u holda (71) tenglamaning yechimi

$$u(\xi) = \frac{k^2(\lambda^2 - 4\mu)}{2} \left(\frac{c_1 \operatorname{sh} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi + c_2 \operatorname{ch} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi}{c_1 \operatorname{ch} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi + c_2 \operatorname{sh} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi} \right)^2 - \frac{\lambda^2 k^2}{2} + 2k^2 \mu,$$

bo'ladi, agar $(\lambda^2 - 4\mu) < 0$ bo'lsa u holda yechim

$$u(\xi) = \frac{k^2(4\mu - \lambda^2)}{2} \left(\frac{-c_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi + c_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi}{c_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi + c_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi} \right)^2 - \frac{\lambda^2 k^2}{2} + 2k^2 \mu,$$

bo'ladi, bu yerda $\xi = kx - k^3(\lambda^2 - 4\mu)t + k \int_0^t \gamma(\tau) u(0, \tau) d\tau + \Omega^0$,

agar $(\lambda^2 - 4\mu) = 0$ bo'lsa u holda yechim

$$u(x, t) = \frac{2k^2 c_2^2}{(c_1 + \xi c_2)^2},$$

ko'rinishda bo'ladi, bu yerda $\xi = kx + k \int_0^t \gamma(\tau) u(0, \tau) d\tau + \Omega^0$; c_1, c_2 va Ω^0

ixtiyoriy konstantalar.

Uchinchi bobning to'rtinchi paragrafida yuklangan modifisirlangan Korteveg-de Friz tenglamasi aniq yechimini topish algoritmi ko'rsatilgan.

Quyidagi

$$q_t - 6q^2 q_x + q_{xxx} - \gamma(t) q(0, t) q_x = 0, \quad t \geq 0, \quad x \in \mathbb{R}, \quad (72)$$

yuklangan modifisirlangan Korteveg-de Friz tenglamasi berilgan bo'lsin. Bu yerda $\gamma(t)$ oldindan berilgan uzluksiz funksiya. (G' / G) usulini (72) tenglamaga qo'llash orqali biz uch xil turdagi yechimini olamiz. Bu quyidagi teoramada ifodalangan.

6-teorema. Agar $(\lambda^2 - 4\mu) > 0$ bo'lsa u holda (72) tenglamaning yechimi

$$q(\xi) = \frac{k\sqrt{\lambda^2 - 4\mu}}{2} \left(\frac{c_1 \operatorname{sh} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi + c_2 \operatorname{ch} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi}{c_1 \operatorname{ch} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi + c_2 \operatorname{sh} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi} \right),$$

bo'ladi, bu yerda $\xi = kx + \frac{k^3(\lambda^2 - 4\mu)}{2} t + k \int_0^t \gamma(\tau) q(0, \tau) d\tau + \Omega^0$,

agar $(\lambda^2 - 4\mu) < 0$ bo'lsa u holda yechim

$$q(\xi) = \frac{k^2(4\mu - \lambda^2)}{2} \left(\frac{-c_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi + c_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi}{c_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi + c_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi} \right)^2 - \frac{\lambda^2 k^2}{2} + 2k^2 \mu,$$

bo'ladi, bu yerda $\xi = kx - k^3(\lambda^2 - 4\mu)t + k \int_0^t \gamma(\tau)q(0, \tau)d\tau + \Omega^0$,

agar $(\lambda^2 - 4\mu) = 0$ bo'lsa u holda yechim

$$q(\xi) = \frac{2k^2 c_2^2}{(c_1 + \xi c_2)^2},$$

ko'rinishda bo'ladi, bu yerda $\xi = kx + k \int_0^t \gamma(\tau)q(0, \tau)d\tau + \Omega^0$; c_1 , c_2 va Ω^0 ixtiyoriy konstantalar.

Muallif o'zining ilmiy rahbari, f.-m.f.d. O'razboev G'ayrat O'razalievichga doimiy e'tibori hamda mazkur dissertatsiya natijalarini muhokamasidagi qimmatli maslahatlari uchun samimiy minnatdorchiligini bildiradi.

XULOSA

Dissertatsiyaning birinchi bobida Dirak operatori uchun sochilish nazariyasining teskari masalasi va nochiziqli evolyutsion tenglamalarni yechishning to'g'ri usullari haqida zaruriy malumotlar keltirilgan.

Dissertatsiyaning ikkinchi bobi yuklangan nochiziqli Schredinger tenglamasini Dirak operatori uchun sochilish nazariyasining teskari masala usulidan foydalanib integrallash va Gelfand-Levitan-Marchenko integral tenglamasining sonli yechimlarini topishga bag'ishlangan.

Dissertatsiyaning uchinchi bobida to'g'ri usullardan foydalangan holda moslangan manbali nochiziqli Schredinger tenglamasi, yuklangan Schredinger tenglamasi, yuklangan Korteveg-de Friz tenglamasi, yuklangan modifisirlangan Korteveg-de Friz tenglamalarini aniq yechimlari topish ko'rsatilgan.

Tadqiqotning asosiy natijalari quyidagilardan iborat:

tez kamayuvchi funksiyalar sinfida yuklangan nochiziqli Shredinger tenglamasi to'la integrallanuvchi bo'lishi isbotlangan;

tez kamayuvchi funksiyalar sinfida yuklangan nochiziqli Shredinger tenglamasiga qo'yilgan Koshi masalasi yechimining yagonaligi isbotlangan;

Gelfand-Levitan-Marchenko integral tenglamalar sistemasining sonli yechimlari ko'rsatilgan;

moslangan manbali nochiziqli Shredinger tenglamasining soliton yechimlari topilgan;

yuklangan nochiziqli Shredinger tenglamasining soliton yechimlari topilgan;

yuklangan Korteveg-de Friz tenglamasining aniq yechimlari topilgan;

yuklangan modifisirlangan Korteveg-de Friz tenglamasining aniq yechimlari topilgan.

**SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREE
PhD.03/30.12.2019.FM.55.01 URGENCH STATE UNIVERSITY**

URGENCH STATE UNIVERSITY

RAKHIMOV ILKHAM DAVRONBEKOVICH

**DIRECT AND INVERSE METHODS FOR SOLVING LOADED
NONLINEAR EVOLUTION EQUATIONS**

01.01.02 – Differential Equations and Mathematical Physics

**ABSTRACT OF DISSERTATION OF THE DOCTOR OF
PHILOSOFHY (PhD) ON PHYSICAL AND MATEMATICAL SCIENCES**

Urgench – 2022

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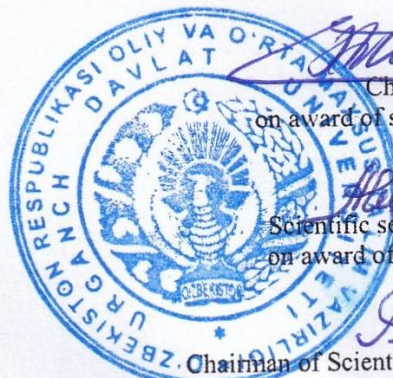
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Council on award of scientific degree, Dr.ph.m.s.

INTRODUCTION (abstract of the PhD thesis)

The actuality(relevance) and demand of the theme of the dissertation. Mathematical models representing nonlinear differential equations with particular derivatives in some cases describe many scientific and practical investigations carrying out on a global scale. A class of equations with special solutions, solitons are stood out among this type of nonlinear differential equations. Nonlinear equations are one of the main equations, such as the Korteweg-de Vries equation, the nonlinear Schrödinger equation, and the modified Korteweg-de Vries equation in the theory of solitons. This kind of equations are very significant in the study of quantum mechanics, nonlinear optics, and the theory of waves in deep water. Thus, the issue of integrating the nonlinear Schrödinger equation, the Korteweg-de Vries equation, and the modified Korteweg-de Vries equation remains one of the important tasks of modern mathematical physics.

At a present, investigations in the theory of nonlinear waves, including the motion of fluids in flexible pipes, optical solitons are applied in the field of telecommunication technology. Real physical systems are characterized by self-consistent source equations, which are a modification of the classical equations. Besides, the effecting forces on physical systems are limited only during a certain period of time so that real models are brought to study the equations of a certain class of functions on spatial variables. The study of self-consistent source and loaded nonlinear evolution equations are considered as one of the priority directions of science, including nonlinear optics, plasma physics, hydrodynamics, and other fields. In this regard, the study of the self-consistent source and loaded nonlinear Schrödinger equation and the loaded Korteweg-de Vries equation and the modified Korteweg-de Vries equation are targeted scientific studies.

In our country, special attention is focused on fundamental research², which have practical implementation. In particular, the analyses of research are the main area in the theory of nonlinear waves, the spectral theory of differential operators, scattering theory of the differential operators, and problems of mathematical physics. As a result, significant works have been done for the complete integrability of the nonlinear evolution equations of mathematical physics using direct and inverse spectral problems. Scientific research appointed as the main tasks and activities in the field of mathematics, physics, and modern methods of mathematical physics. Ensuring the execution of these activities have great importance in the development of the theory of inverse spectral problems and the direct methods, the integration of the nonlinear evolution equations with self-consistent sources and loaded nonlinear evolution equations.

This dissertation, to a certain extent, serves the implementation of the tasks outlined in the Decrees of the President of the Republic of Uzbekistan No.-RP-2909 dated April 20, 2017 “On measures for the further development of the higher education system”, No. RP-2789 dated February 17, 2017 “On measures for the

² Decree of Cabinet of Ministries of the Republic of Uzbekistan at the 2017 year 18 may “On measures on the organization of activities of the first created scientific research institutions of the Academy of Science of the Republic Uzbekistan”.

further improvement of activities of the Academy of Sciences, organization, management and financing of research activities” and No.-DP-4947 dated February 7, 2017 “On the strategy of action for the further development of the Republic of Uzbekistan” and No.-DP-4387 from July 9, 2019 “On measures to further development of mathematical education and science, as well as root improvement of the activities of the Uzbekistan Academy of Sciences the V.I. Romanovsky Institute of Mathematics”, as well as in other legal regulations related to the basic sciences.

Connection of the research to the priority areas of the development of science and technology of the Republic. This study was carried out in accordance with the priority area of the development of science and technology in the Republic of Uzbekistan. “Mathematics, mechanics and computer science”.

The degree of scrutiny of the problem. In 1967, C.S. Gardner, J.M. Green, M. Kruskal, and R. Miura showed that the solution of the Korteweg - de Vries equation can be obtained for all “rapidly decreasing” initial conditions, that is, conditions that in a certain way vanish as the coordinate tends to infinity. This method is called the method of the inverse scattering problem, since it essentially uses the solution of the problem of reconstructing the potential of the Sturm-Liouville operator on the entire axis from the scattering data. P. Lax showed that the method used by these scientists was universal. In 1972, V.E. Zakharov and A.B. Shabat showed the complete integrability of the nonlinear Schrödinger equation via the inverse scattering method. In 1972, M. Wadati, using the ideas of the inverse scattering problems, proposed a solution of the modified Korteweg - de Vries equation. In 1988, the fundamental works of V.K. Melnikov on nonlinear evolution equations with a self-consistent source, using the inverse scattering method, the Korteweg - de Vries equations with a self-consistent source were integrated in the class of “rapidly decreasing” functions. We also note that in the work of C. Claude, J. Leon and A. Latifi a specific physical problem was presented, which was reduced to solving the Korteweg - de Vries equation with a source. A.B. Khasanov and G.U. Urazboev studied in the class of “step-like” functions, A.B. Khasanov and U.A. Khoitmetov in the class of complex-valued functions, and in the class of periodic functions by A.B. Khasanov and A.B. Yakhshimuratov. The modified Korteweg-de Vries equation with the self-consistent sources in various classes of functions were investigated by A.B. Khasanov, G.U. Urazboev, K.A. Mamedov, M.M. Khasanov. The nonlinear Schrödinger equation with the self-consistent sources in various classes of functions were investigated by A.B. Khasanov, A.A. Reyimberganov.

Besides, there are direct methods to solve nonlinear evolution equations. In 1971, Ryogo Hirota published an article giving a new method called “the Hirota direct method” to find the exact solution of the Korteweg-de Vries equation for multiple collisions of solitons. In his successive articles, he dealt also with many other nonlinear evolution equations such as the modified Korteweg-de Vries, nonlinear Schrödinger equations. In addition, other researchers have studied evolutionary equations using the Hirota direct method, including Yong Li Sun, Wen Xiu Ma, Jian Ping Yu, Abdul Majid Wazwaz, Yichao Ye, Lihong Wang,

Zhaowei Chang, Jingsong He, Metin Gürses, Aslı Pekcan, Wen Xiu Ma, Sachin Kumar, Brij Mohan, Yi Zhang, Li-gang Jin, C. T. Lee, C., C. Lee, Yi Zhang, Yineng Lv, Ling-ya Ye, Hai-qiong Zhao, Fucui You, Jiao Zhang, Jianbing Zhang, Li Li, Chaonan Duan, Fajun Yu.

In recent years, extensive research has been conducted on nonlinear evolution equations with self-consistent source using the Hirota method. D. J. Zhang (2002) showed that N-soliton solutions for the modified Korteweg-de Vries equation with self-consistent source.

Alternatively, the (G'/G) - expansion method is also effective in finding traveling wave solutions of nonlinear evolution equations. The advantage of the (G'/G) - expansion method is that it allows obtaining more general solutions with some free parameters. In 2008, M. Wang, X. Li, J. Zhang gave an algorithm for finding exact solutions of nonlinear evolution equations using the (G'/G) - expansion method.

The connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. The dissertation has been executed according to the planned theme of the scientific research work of the “Applied mathematics and mathematical physics” department of the Urgench State University and in the frame of the scientific research project on the theme F-4-61 “Integration of the nonlinear evolution equations with a self-consistent source via inverse problem method” (2019-2021yy).

The aims of research work are: to study of the soliton solutions for a nonlinear Schrödinger equation with a self-consistent source, loaded nonlinear Schrödinger equation, loaded Korteweg-de Vries equation and loaded modified Korteweg-de Vries equations.

Research problems are:

application of the inverse scattering theory for the Dirac system with a rapidly decreasing potential to the integration of the loaded nonlinear Schrödinger equation;

application of numerical methods to the system of Gelfand-Levitan-Marchenko integral equation of scattering theory;

application of the Hirota direct method for the nonlinear Schrödinger equation with self-consistent source;

application of the Hirota direct method for the loaded nonlinear Schrödinger equation;

application of the (G'/G) – expansion method for the loaded Korteweg–de Vries equation;

application of the (G'/G) – expansion method for the loaded modified Korteweg-de Vries equation.

The research objects are: nonlinear Schrödinger equation with self-consistent source, loaded Korteweg-de Vries equation, loaded modified Korteweg–de Vries equation, loaded nonlinear Schrödinger equation.

The research subjects are: Hirota direct method, (G'/G) – expansion method, inverse scattering problem for the nonlinear evolution equations with a self-consistent source and the nonlinear evolution equations with a loaded term.

Research methods: The research used the methods of mathematical physics, spectral analysis, functional analysis, theory of functions of complex variables and methods of solving differential equations.

Scientific novelty of research work consists of the followings:

proved the complete integrability loaded nonlinear Schrödinger equation in the class of rapidly decreasing functions;

proved the uniqueness theorem the Cauchy problems for the loaded nonlinear Schrödinger equation in the class of rapidly decreasing functions;

shown numerical solutions of the system Gelfand-Levitan-Marchenko integral equations;

found the soliton solutions of the nonlinear Schrödinger equation with self-consistent source in the class of rapidly decreasing functions;

found the soliton solutions of the loaded nonlinear Schrödinger equation in the class of rapidly decreasing functions;

shown exact solutions of the loaded Korteweg-de Vries equation; shown exact solutions of the loaded modified Korteweg-de Vries equation.

The practical results of the study consist of applying algorithms to solve nonlinear Schrödinger equation with the self-consistent source, loaded Korteweg-de Vries equation, loaded modified Korteweg-de Vries equation, loaded nonlinear Schrödinger equation, Cauchy problems for the loaded nonlinear Schrödinger equation and numerical solution of the system Gelfand-Levitan-Marchenko integral equation.

Reliability of the research results is justified using the methods of mathematical physics, spectral analysis, functional analysis and the theory of functions of complex variables in solving inverse spectral and scattering problems for differential operators with rapidly decreasing coefficients and has shown their application for solving nonlinear evolution equations, as well as the rigor of mathematical reasoning and proof.

Scientific and practical significance of the research results. The scientific significance of the research results lies in the fact that the obtained scientific results can be used in the spectral theory of linear operators, in the problems of the dynamics of thin elastic rods, phonons in anharmonic lattices, traffic congestions, hyperbolic surfaces and ionic acoustic solitons.

The practical significance of the dissertation is that the obtained scientific results can be used in mathematical physics such as integrating nonlinear evolution equations in the various classes of functions.

Implementation of research results. The obtained results in the dissertation were applied in the following areas:

the method of calculating the main integral equation of inverse scattering theory the numerical solutions of the system Gelfand-Levitan-Marchenko integral equations was used in the project “Slovak Research and Development Agency under contract No. APVV-18-0308” and “Slovak Grant Agency VEGA No.

1/0358/20 and No. 2/0127/20". (reference of the University of Comenius of the state of Slovakia on June 24, 2022);

in the project "Ordinary and Functional Differential Equations, code MTM2016-75140-P", supported by the Ministry of Economy and Competitivity of Spain at the University of Santiago de Compostela (Spain, University of Santiago de Compostela reference dated June 27, 2022), the nonlinear Schrödinger equation was used to find numerical analytical solutions.

Approbation of the research results. The results were discussed at 6 scientific and practical conferences, including 4 international and 2 national ones.

Publication of the research results. According to the theme of the dissertation, 14 scientific papers were published, such as 3 articles were published in the international journals, which 2 are indexed in Scopus, and 5 articles in the national scientific journals included in the list of scientific journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for the defense of theses of the Doctor of Philosophy.

The structure and volume of the dissertation. The dissertation consists of three chapters, conclusion and list of the used literatures. The volume of the dissertation is 81 pages.

THE MAIN CONTENT OF THE THESIS

The introduction part of the thesis includes the actuality and the demand of the research, the relevance of the research to the priority areas of science and technology, the review of foreign research on the topic, the degree of scrutiny of the problem. Also the introductory part contains the aim of the research work, research problems, the object and the subject of the research, scientific novelty and practical results, theoretical, practical significance and the statement of the research results, published works and the information on the structure of the thesis.

In the first chapter of the theses, titled "**Scattering theory for Dirac system and direct methods for nonlinear evolution equations**", it was studied direct and inverse scattering problems for the Dirac operator on the entire axis and direct methods for solving nonlinear evolution equation.

Consider the system of linear equations on the real line ($-\infty < x < \infty$)

$$Lv = \xi v \tag{1}$$

where $v = (v_1, v_2)^T$ is vector function and

$$L = i \begin{pmatrix} \frac{d}{dx} & -u(x) \\ -u^*(x) & -\frac{d}{dx} \end{pmatrix},$$

with function $u(x)$ satisfying the conditions

$$\int_{-\infty}^{\infty} (1 + |x|) |u(x)| dx < \infty. \tag{2}$$

Under condition (2), system of equations (1) has Jost solutions with the following asymptotics

$$\varphi(x, \xi) \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\xi x}, \quad \bar{\varphi}(x, \xi) \sim \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{i\xi x} \quad \text{as } x \rightarrow -\infty, \quad (3)$$

$$\psi(x, \xi) \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-i\xi x}, \quad \bar{\psi}(x, \xi) \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{i\xi x} \quad \text{as } x \rightarrow \infty. \quad (4)$$

(Note that $\bar{\varphi}$ ($\bar{\psi}$) is not complex conjugation of φ (ψ)).

Jost solutions $\varphi(x, \xi)$ and $\bar{\psi}(x, \xi)$ ($\bar{\varphi}(x, \xi)$ and $\psi(x, \xi)$) admit an analytic continuation in ξ into the upper (lower) half-plane $\text{Im}\xi > 0$ ($\text{Im}\xi < 0$). For any $\xi \in (-\infty, \infty)$ the pairs of functions $\{\varphi(x, \xi), \bar{\varphi}(x, \xi)\}$ and $\{\psi(x, \xi), \bar{\psi}(x, \xi)\}$ are the pairs of linearly independent solutions of (1), and holds the equality

$$\begin{cases} \varphi = a(\xi)\bar{\psi} + b(\xi)\psi, \\ \bar{\varphi} = -\bar{a}(\xi)\psi + \bar{b}(\xi)\bar{\psi}. \end{cases} \quad (5)$$

where the functions $a(\xi)$ and $b(\xi)$ are independent of x and

$$a(\xi) = W\{\varphi, \psi\} \equiv \varphi_1\psi_2 - \varphi_2\psi_1. \quad (6)$$

Moreover, for real ξ

$$a(\xi)\bar{a}(\xi) + b(\xi)\bar{b}(\xi) = 1. \quad (7)$$

Hence, the function $a(\xi)$ ($\bar{a}(\xi)$) can be analytically extended in upper (lower) half-plane $\text{Im}\xi > 0$ ($\text{Im}\xi < 0$). The function $a(\xi)$ has the asymptotic

$$a(\xi) = 1 + O\left(\frac{1}{|\xi|}\right) \quad \text{as } |\xi| \rightarrow \infty, \quad \text{Im}\xi \geq 0. \quad \text{Besides, in the half-plane } \text{Im}\xi > 0$$

($\text{Im}\xi < 0$) the function $a(\xi)$ ($\bar{a}(\xi)$) has a finite number of zeros at the points ξ_k ($\bar{\xi}_k$) ($k = 1, 2, \dots, N$), and these points are the eigenvalues of the operator L .

Note that the vector functions

$$h_n(x) = \frac{\frac{d}{d\xi}(\varphi - C_n\psi)\Big|_{\xi = \xi_n}}{\dot{a}(\xi_n)}, \quad n = 1, 2, \dots, N \quad (8)$$

are solutions to equations $Lh_n = \xi_n h_n$. According to equality $a(\xi) = W\{\varphi, \psi\}$, we obtain the following asymptotics

$$\begin{aligned} \psi &\sim a(\xi) \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\xi x} \quad \text{as } x \rightarrow -\infty, \\ \varphi &\sim a(\xi) \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\xi x} \quad \text{as } x \rightarrow \infty, \end{aligned}$$

which are valid for $\text{Im}\xi > 0$. From these estimates and equality (8) we obtain the following asymptotics

$$h_n \sim -C_n \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\xi_n x} \quad \text{as } x \rightarrow -\infty, \quad h_n \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\xi_n x} \quad \text{as } x \rightarrow \infty. \quad (9)$$

In particular

$$W\{\varphi_n, h_n\} \equiv \varphi_{n1} h_{n2} - \varphi_{n2} h_{n1} = -C_n, \quad (10)$$

where $\varphi_n \equiv \varphi(x, \xi_n)$, $n = 1, 2, \dots, N$.

Definition 1. The set $\{r(\xi) \equiv \frac{b(\xi)}{a(\xi)}, \xi_k, C_k\}$, $\xi \in R^1, \text{Im } \xi_k > 0, k = \overline{1, N}$ is called the scattering data associated with the equation (1).

Definition 2. The direct scattering problem is to find the scattering data via the given potential $u(x)$.

Definition 3. The inverse scattering problem is to find the potential $u(x)$ of the equation (1) via the given scattering data.

For the functions ψ , $\bar{\psi}$ the following integral representations are valid

$$\psi = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\xi x} + \int_x^\infty K(x, s) e^{i\xi s} ds, \quad (11)$$

$$\bar{\psi} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\xi x} + \int_x^\infty \bar{K}(x, s) e^{-i\xi s} ds, \quad (12)$$

where $K(x, s)$, $\bar{K}(x, s)$ are two-component vectors, i.e.

$$K(x, s) = \begin{pmatrix} K_1(x, s) \\ K_2(x, s) \end{pmatrix}.$$

In representations (11), (12), the kernels $K(x, s)$ and $\bar{K}(x, s)$ do not depend on ξ and is related to $u(x)$ by means of the equality

$$u(x) = -2K_1(x, x). \quad (13)$$

The kernels $K(x, s)$ and $\bar{K}(x, s)$ for $y > x$ are the solution of the Gelfand-Levitan-Marchenko integral equations

$$\bar{K}(x, y) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} F(x+y) + \int_x^\infty K(x, s) F(s+y) ds = 0,$$

$$K(x, y) - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \bar{F}(x+y) - \int_x^\infty \bar{K}(x, s) \bar{F}(s+y) ds = 0,$$

where

$$F(x) = \frac{1}{2\pi} \int_{-\infty}^\infty \frac{b(\xi)}{a(\xi)} e^{i\xi x} d\xi - i \sum_{j=1}^N C_j e^{i\xi_j x}, \quad \bar{F}(x) = \frac{1}{2\pi} \int_{-\infty}^\infty \frac{\bar{b}(\xi)}{a(\xi)} e^{-i\xi x} d\xi + i \sum_{j=1}^N \bar{C}_j e^{-i\xi_j x}.$$

In the next paragraph of the first chapter provides the necessary information about Hirota direct method and (G'/G) - expansion method of finding exact solutions of nonlinear evolution equations.

Let us introduce with the Hirota direct method. Let

$$F[u] = F(u, u_x, u_t, \dots) = 0$$

be a nonlinear partial differential or difference equation. As the first step we transform $F[u]$ to a quadratic form in the dependent variables by using a transformation

$$u = T[f(x, t, \dots), g(x, t, \dots)].$$

We name this new form as bilinear form of $F[u]$. We should denote that we could not find such a transformation for some equations. We should also note some integrable equations like the KdV equation can be transformed to a single bilinear equation, however many of them like mKdV and nLS equations can only be formed as combination of bilinear equations. Then we introduce the Hirota D-operator which makes the Hirota method very productive.

Definition 4. $S : C^n \rightarrow C$ is a space of differentiable functions. Then the Hirota D -operator $D : S \times S \rightarrow S$ is expressed as

$$[D_x^{m_1} D_t^{m_2} \dots] \{f \cdot g\} = [(\partial_x - \partial_{x'})^{m_1} (\partial_t - \partial_{t'})^{m_2} \dots] f(x, t, \dots) \times g(x', t', \dots) \Big|_{x'=x, t'=t, \dots}, \quad (14)$$

Here x, t, \dots are independent variables and $m_j, j = 1, 2, \dots$ are positive integer.

We express $F[u]$ in the bilinear form by using D -operator as a polynomial. This is called polynomial $P(D)$.

Definition 5. It is said that it is written in Hirota bilinear form as following

$$\sum_{\alpha, \beta=1}^m P_{\alpha\beta}^\eta(D) f^\alpha f^\beta = 0, \quad \eta = 1, \dots, r, \quad (15)$$

here for m, r and linear operators $P_{\alpha\beta}^\eta(D), f^i$ are new dependent variables.

Now let us state and prove some propositions and corollaries on $P(D)$.

Proposition 1. If f and g are differentiable functions, then

$$P(D)\{f \cdot g\} = P(-D)\{f \cdot g\}. \quad (16)$$

Proposition 2. For the functions f and $g = 1$ followings are suitable

$$P(D)\{f \cdot 1\} = P(\partial)f, \quad P(D)\{1 \cdot f\} = P(-\partial)f. \quad (17)$$

Proposition 3. Two exponential functions e^{θ_1} and e^{θ_2} , where $\theta_j = k_j x + \dots r_j z + l_j y + \alpha_j$ and $k_j, \dots, r_j, l_j, \alpha_j$ are constants for $j = 1, 2$. We have

$$P(D)\{e^{\theta_1} \cdot e^{\theta_2}\} = P(k_1 - k_2, \dots, r_1 - r_2, l_1 - l_2) e^{\theta_1 + \theta_2}. \quad (18)$$

Proposition 4. If we have a system such that $P(D)\{a \cdot a\} = 0$, where is any non-zero constant then by proposition 3 we have $P(0, 0, \dots, 0) = 0$.

Remark. If we consider $P(D)\{f \cdot f\}$, we may assume P is even since the odd terms cancel due to the ant symmetry of the D -operator i.e. we have

$D_{x_1}^{m_1} D_{x_2}^{m_2} \dots D_{x_k}^{m_k} \{f \cdot f\} = 0$ identically satisfied if $\sum_{j=1}^k m_j = \text{odd}$. For instance, as

simple examples we clearly have

$$D_x \{f \cdot f\} = f_x f - f f_x = 0,$$

$$D_t D_x^2 \{f \cdot f\} = f_{xt} f - f_{xx} f_t - f_{xt} f_x + f_x f_{xt} - f_{xt} f_x + f_x f_{xt} + f_t f_{xx} - f f_{xt} = 0.$$

In this paragraph, we present the main steps of the (G' / G) -expansion method.

Let us given a nonlinear partial differential equation in the form below

$$F(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \dots) = 0, \quad (19)$$

where $u = u(x, t)$ is unknown function, F is a nonlinear function of $u(x, t)$ and its partial derivatives. Now we give the main steps of the (G'/G) - expansion method:

Step 1. We use the travelling wave transformation in the following form:

$$u(x, t) = u(\xi), \quad \xi = kx + \Omega(t), \quad (20)$$

where k is a parameter and $\Omega(t)$ is a continuous function which is dependent on t . The representation of the solution in the form (20) allows us to reduce equation (19) to an ordinary differential equation

$$P(u, u', u'', u''', \dots) = 0, \quad (21)$$

where $u' = \frac{du(\xi)}{d\xi}$, $u'' = \frac{d^2u(\xi)}{d\xi^2}$, $u''' = \frac{d^3u(\xi)}{d\xi^3}$, \dots

Step 2. Let us suppose that the solution of the equation (21) can be represented as a polynomial of $\left(\frac{G'}{G}\right)$:

$$u(\xi) = \sum_{j=0}^m a_j \left(\frac{G'}{G}\right)^j, \quad (22)$$

where G' is the derivative of $G(\xi)$ with respect to ξ , a_j ($j=0, 1, 2, \dots, m$) are constants that can be determined later, m - positive integer number determined from the balance of higher order derivatives and the function $G = G(\xi)$ satisfies the following linear ordinary differential equation of the second order:

$$G'' + \lambda G' + \mu G = 0, \quad (23)$$

where λ, μ are arbitrary constants.

Step 3. We can use (23) when we substitute (22) into equation (21) and gain a system of equations for determining $k, \Omega(t), a_j, j=1, 2, \dots, m$.

Step 4. Substituting the obtained values $k, \Omega(t), \lambda, \mu, a_j, j=1, 2, \dots, m$, into (22) and using (23), we can find exact solutions of equation (19).

In the second chapter of the dissertation, titled “**Integration of the loaded nonlinear Schrödinger equation via inverse scattering method**”, it was proved the completely integrability of the loaded nonlinear Schrödinger equation in the class of rapidly decreasing functions and found numerical solution of the main integral equation Gelfand-Levitan-Marchenko of the inverse scattering theory.

In the first part of the second chapter we have considered integration of the loaded nonlinear Schrödinger equation in the class of rapidly decaying functions.

We consider the loaded nonlinear Schrödinger equation of the following problem

$$iu_t + 2|u|^2 u + u_{xx} - i\gamma(t)(u(0, t) + u^*(0, t))u_x = 0, \quad (24)$$

$$u(x, 0) = u_0(x), \quad (25)$$

where the real function $u_0(x)$ satisfy the following properties:

1. $\int_{-\infty}^{\infty} (1+|x|)|u_0(x)|dx < \infty$;
2. The equation

$$L(0)y \equiv i \begin{pmatrix} \frac{d}{dx} & -u_0(x) \\ -u_0^*(x) & -\frac{d}{dx} \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \xi \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}, \quad x \in R \quad (26)$$

can have N number of simple eigenvalues and does not have spectral singularities. Here, the function $u_0^*(x)$ is a complex conjugation of $u_0(x)$.

The main goal of this paragraph is to study the integration of the loaded nonlinear Schrödinger equation in the class of $u(x,t)$ function, which is sufficiently smooth and tends to its limits rapidly enough when $x \rightarrow \pm\infty$ and satisfies the condition

$$\int_{-\infty}^{\infty} \left((1+|x|)|u_0(x)| + \sum_{j=1}^3 \left| \frac{\partial^j u(x,t)}{\partial x^j} \right| \right) dx < \infty, t \geq 0$$

via inverse scattering problem.

Theorem 1. If $u(x,t)$ function is solution of the problem (24)-(26) then it is solution unique.

Theorem 2. If the function $u(x,t)$ is a solution the problem (24)-(26), then the scattering data for the operator $L(t)$ satisfy the relations

$$\begin{aligned} \frac{dr^+}{dt} &= \left(4i\xi^2 - 2\xi\gamma(t)(u(0,t) + u^*(0,t)) \right) r^+, \quad \text{Im } \xi = 0, \\ \frac{dC_n}{dt} &= \left(4i\xi_n^2 + 2i\xi_n\gamma(t)(u(0,t) + u^*(0,t)) \right) C_n, \\ \frac{d\xi_n}{dt} &= 0, \quad n = 1, 2, 3, \dots, N. \end{aligned}$$

The obtained relations determine completely the evolution of the scattering data for the system (26), which allows us to find the solution of the problem (24)-(25) by using the Inverse scattering problem method. Using the following algorithm, we can find the solution. Let us given the functions $u_0(x)$

1. Solving the direct scattering problem for the initial $u_0(x)$, we obtain the scattering data $\{r(\xi, 0), \xi_1(0), \xi_2(0), \dots, \xi_N(0), C_1(0), C_2(0), \dots, C_N(0)\}$ of the operator $L(0)$.
2. Using the results of the theorem 2, we find the scattering data $\{r(\xi, t), \xi_1(t), \xi_2(t), \dots, \xi_N(t), C_1(t), C_2(t), \dots, C_N(t)\}$ for $t > 0$.
3. Using the method based on Gelfand-Levitan-Marchenko integral equation, we solve the inverse scattering problem for $L(t)$ and determine $u(x,t)$ by the scattering data $\{r(\xi, t), \xi_1(t), \xi_2(t), \dots, \xi_N(t), C_1(t), C_2(t), \dots, C_N(t)\}$

In the second paragraph of the second chapter, it is shown to find the numerical solutions of the Gelfand-Levitan-Marchenko integral equation.

The coupled system of Gelfand-Levitan-Marchenko integral equations as following

$$\begin{cases} \bar{K}_2(x, y) + \int_0^\infty K_1(x, s+x)F(s+x+y)ds = 0, \\ -K_1(x, y) + \bar{F}(x+y) + \int_0^\infty \bar{K}_2(x, s+x)\bar{F}(s+x+y)ds = 0, \end{cases} \quad (27)$$

where

$$F(x) = \frac{1}{2\pi} \int_{-\infty}^\infty R(\xi)e^{i\xi x} d\xi - i \sum_{k=1}^N c_k e^{i\xi_k x} \quad (28)$$

are solved by the Nystrom method. As the Gelfand-Levitan-Marchenko kernel in (27) depends on $s+x$ and $s+x+y$, because we create grid points as the following:

$$\begin{aligned} s_j &= (j-1)\tau, \quad \beta_{k+j} = x_k + s_j, \quad \alpha_{k+j+p} = \beta_{k+j} + \beta_{p+k}, \\ \alpha_{k+p} &= x_k + \beta_{p+k}, \quad k = 1, 2, \dots, N_x, \quad j = 1, 2, \dots, N_y. \end{aligned} \quad (29)$$

We use the same grid points to approximate the integrals by the composite Simpson's quadrature rule. According to grid points there holds the system of linear equation

$$\begin{cases} K_2^*(x_k, \beta_{k+p}) + \sum_{j=1}^{N_y} d_j K_1(x_k, \beta_{k+j})F(\alpha_{k+j+p}) = 0, \\ \sum_{j=1}^{N_y} d_j K_2^*(x_k, \beta_{k+j})F^*(\alpha_{k+j+p}) - K_1(x_k, \beta_{k+p}) = -F^*(\alpha_{k+p}), \end{cases} \quad (30)$$

where $\{d_i\}_{i=0}^{N_y} = \frac{\tau}{3}\{1, 4, 2, 4, 2, 4, \dots, 1\}$, τ is step size of y .

In matrix notation the previous system (30) takes the form

$$\begin{bmatrix} I_{N_y} & HD \\ H^*D & -I_{N_y} \end{bmatrix} \begin{bmatrix} k_2 \\ k_1 \end{bmatrix} = \begin{bmatrix} 0 \\ -f_{N_y} \end{bmatrix}, \quad (31)$$

where $D = \text{diag}\{d_j\}_{j=1}^{N_y}$, $I_{N_y} = \text{diag}\{1\}_{j=1}^{N_y}$,

$$H = (F(\alpha_{k+j+p}))_{p,j=1}^{N_y}, \quad f_{N_y} = (F(\alpha_{k+p}))_{p=1}^{N_y}, \quad (32)$$

and the unknown vector-columns are

$$k_2 = (K_2(x_k, \beta_{k+p}))_{p=1}^{N_y}, \quad k_1 = (K_1(x_k, \beta_{k+p}))_{p=1}^{N_y}. \quad (33)$$

System (31) can be represented by the following form

$$\begin{cases} (DH * DHD + D)k_1 = Df_{N_y}, \\ k_2 = -HDk_1. \end{cases}$$

Here the matrix $DH * DHD + D$ is symmetric and positive definite, so the system can be easily solved by the conjugate gradient method. A square matrix H of order $N_y \times N_y$ is called a Hankel matrix, as $(H)_{ij} = F(\alpha_{k+i+j})$. Note that the potential we want to obtain, as well as the associated $u(x)$ are defined by the first component of k_1 as (13) and (33) show.

In the third chapter of the thesis, titled “**Direct methods for the solution of the nonlinear evolution equations**”, it was given the algorithm to find the exact soliton solutions of the nonlinear Schrödinger equation with self-consistent source, loaded nonlinear Schrödinger equation in the class rapidly decreasing functions, loaded Korteweg-de Vries and loaded modified Korteweg-de Vries equation by using direct methods.

In the first paragraph of the third chapter, it was found soliton solutions of the nonlinear Schrödinger equation with a self-consistent source in the class of rapidly decreasing functions by using Hirota direct method.

We consider the integration of the following system of equations

$$iu_t + 2|u|^2 u + u_{xx} = 2i \sum_{j=1}^N (\varphi_{1j}^2 - \bar{\varphi}_{2j}^2), \quad (34)$$

$$\begin{cases} \varphi_{1j,x} = i\xi_j \varphi_{1j} - u \varphi_{2j}, \\ \varphi_{2j,x} = -i\xi_j \varphi_{2j} + \bar{u} \varphi_{1j}, \end{cases} \quad j = 1, 2, \dots, N, \quad (35)$$

where the bar means complex conjugation and ξ_j , $j = 1, 2, \dots, N$ are the eigenvalues.

We assume that the solution $u(x, t)$ of the system (34) - (35) exists possessing the required smoothness and tends to its limits sufficiently rapidly as $|x| \rightarrow \infty$, i.e., for all $t \geq 0$ satisfies the condition

$$\int_{-\infty}^{\infty} (1 + |x|) |u(x, t)| dx + \int_{-\infty}^{\infty} \sum_{k=1}^2 \left| \frac{\partial^k u(x, t)}{\partial x^2} \right| dx < \infty. \quad (36)$$

Under the condition shown below the system of equations (35) has a finite number of eigenvalues. In general, these eigenvalues can be multiples. Here, we assume that all the eigenvalues are simple and their numbers are equal to N . We also assume that the eigenfunctions $\Phi_j = (\varphi_{1j}, \varphi_{2j})^T$ corresponding to this eigenvalues satisfy the following normalizing conditions

$$\int_{-\infty}^{\infty} \varphi_{1j} \varphi_{2j} dx = \beta_j^2(t), \quad j = 1, 2, \dots, N. \quad (36')$$

Here $\beta_j(t)$, $j = 1, 2, \dots, N$ are given and the continuous functions of t .

We will find the soliton solution of the nonlinear Schrödinger equation with self-consistent sources by using of Hirota's method. With the help of the dependent variable transformations

$$u = \frac{g}{f}, \quad \varphi_{1j} = \frac{p_j}{f}, \quad \varphi_{2j} = \frac{h_j}{f}, \quad j = 1, 2, \dots, N, \quad (37)$$

the system (34)-(35) can be transformed into the bilinear forms

$$(iD_t + D_x^2)g \cdot f = 2i \sum_{j=1}^N (p_j^2 - \bar{h}_j^2), \quad (38)$$

$$D_x^2 f \cdot f = 2g \cdot \bar{g}, \quad (39)$$

$$\begin{cases} D_x p_j \cdot f = -i\xi_j p_j f + g h_j, \\ D_x h_j \cdot f = i\xi_j h_j f - \bar{g} p_j, \end{cases} \quad (40)$$

where \bar{g} and \bar{h} are the complex conjugation of the functions g and h , respectively and Hirota's bilinear operators D_t and D_x are defined by

$$D_x^m D_t^n g(x,t) \cdot f(x,t) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'} \right)^n g(x,t) f(x',t') \Big|_{x=x', t=t'}. \quad (41)$$

Here, the subscripts of the functions f and g define the order of the partial derivatives with respect to x and t .

Equations (38)-(40) can be solved by introducing the following power series expansions for f , g , p_j and h_j :

$$f = 1 + \chi^2 f^{(1)} + \chi^4 f^{(2)} + \dots, \quad (42)$$

$$g = \chi g^{(1)} + \chi^3 g^{(2)} + \dots, \quad (43)$$

$$p_j = \chi p_j^{(1)} + \chi^3 p_j^{(2)} + \dots, \quad (44)$$

$$h_j = h_j^{(1)}, \quad (45)$$

where χ is a formal expansion parameter. Substituting functions (42)-(45) into equations (38)-(40) and equating coefficients of the same powers of χ to zero can yield the recursion relation for $f^{(k)}$, $g^{(k)}$, $p_j^{(k)}$ and $h_j^{(k)}$, $k=1,2,\dots$. Solving them and we have the solution to the problem (34)-(35) considering transferring (37).

We will give the analytical expression of one-soliton solution (i.e. in the case $N=1$) of the system (34)-(35). According to in the known Hirota's method, we consider for the one-soliton solution of nonlinear Schrödinger equation with self-consistent sources in the form below

$$g = \chi g^{(1)}, \quad f = 1 + \chi^2 f^{(1)}.$$

Using the definition (41) the above (39) equation can be expressed in details. Substituting these expressions into (39) and equating the coefficients of the same powers of χ we have system of equation and solving them as following as

$$g^{(1)} = e^{\eta}, \quad f^{(1)} = e^{\eta + \bar{\eta} + a_{11}}, \quad (46)$$

where $\eta_1 = k_1 x + \gamma_1(t)$, $a_{11} = \ln \frac{1}{(k_1 + \bar{k}_1)^2}$, k_1 is constant and $\gamma_1(t)$ is an arbitrary

function of t . The next step is to find functions p_1 and h_1 in case when one-soliton solution are

$$p_1 = \chi p_j^{(1)}, \quad h_1 = h_j^{(1)}.$$

Based on the above, we collect coefficients of the same power in χ according to the equation (40) and we get system of equation and by solving them we have

$$\begin{aligned} p_1^{(1)} &= e^{(k_1 + \bar{k}_1 - i\xi_1)x + \Omega_1(t)}, \\ h_1^{(1)} &= (k_1 + \bar{k}_1)e^{(\bar{k}_1 - i\xi_1)x + \Omega_1(t) - \gamma_1(t)}, \end{aligned} \quad (47)$$

where $\Omega_1(t)$ is an arbitrary continuous function of t .

Using expressions (46) and (47), we can rewrite the functions f , g , p_1 and h_1 in the following form:

$$\begin{aligned} f &= 1 + e^{\eta_1 + \eta_2 + a_{11}}, \\ g &= e^\eta, \\ p_1 &= e^{(k_1 + \bar{k}_1 - i\xi_1)x + \Omega_1(t)}, \\ h_1 &= (k_1 + \bar{k}_1)e^{(\bar{k}_1 - i\xi_1)x + \Omega_1(t) - \gamma_1(t)}. \end{aligned} \quad (48)$$

We can find $\Omega_1(t)$, $\gamma_1(t)$ functions and k_1 constant by using functions (48) and equation (38)

$$\begin{aligned} \Omega_1(t) &= \ln \beta_1(t) + \frac{1}{2}(\bar{\gamma}_1(t) + a_{11}) + \gamma_1(t), \\ \gamma_1(t) &= -4i\bar{\xi}_1^2 t - 2 \int_0^t \bar{\beta}_1^2(\tau) d\tau + \gamma_1(0), \\ k_1 &= -2i\bar{\xi}_1. \end{aligned} \quad (49)$$

Thus, taking into account (37), (48) and (49) we can write the one-soliton solution of the problem (34)-(35) in the following form

$$\begin{aligned} u(x, t) &= \frac{e^{-2i\bar{\xi}_1 x + \gamma_1(t)}}{1 + e^{(-2i\bar{\xi}_1 + 2i\xi_1)x + \gamma_1(t) + \bar{\gamma}_1(t) + a_{11}}}, \\ \varphi_{11}(x, t) &= \beta_1(t) \frac{e^{(-2i\bar{\xi}_1 + i\xi_1)x + \gamma_1(t) + \bar{\gamma}_1(t) + a_{11}}}{1 + e^{(-2i\bar{\xi}_1 + 2i\xi_1)x + \gamma_1(t) + \bar{\gamma}_1(t) + a_{11}}}, \\ \varphi_{21}(x, t) &= \beta_1(t) \frac{(-2i\bar{\xi}_1 + 2i\xi_1)e^{i\xi_1 x + \bar{\gamma}_1(t) + a_{11}}}{1 + e^{(-2i\bar{\xi}_1 + 2i\xi_1)x + \gamma_1(t) + \bar{\gamma}_1(t) + a_{11}}}. \end{aligned} \quad (50)$$

We find two-soliton solution of the nonlinear Schrodinger equation with self-consistent source (i.e. in the case $N = 2$). We take the functions f , g , p_j and h_j in the following form

$$\begin{aligned} g &= \chi g^{(1)} + \chi^3 g^{(2)}, \quad f = 1 + \chi^2 f^{(1)} + \chi^4 f^{(2)}, \\ p_j &= \chi p_j^{(1)} + \chi^3 p_j^{(2)}, \quad h_j = h_j^{(1)} \end{aligned}$$

By applying the same previous procedure. We can write the solution in the following form

$$u(x, t) = \frac{g^{(1)} + g^{(2)}}{1 + f^{(1)} + f^{(2)}}, \quad (51)$$

$$\varphi_{1j} = \frac{p^{(1)} + p^{(2)}}{1 + f^{(1)} + f^{(2)}}, \quad \varphi_{2j} = \frac{h^{(1)}}{1 + f^{(1)} + f^{(2)}}, \quad j=1,2$$

Where

$$f^{(1)} = e^{\eta_1 + \bar{\eta}_1 + a_{11}} + e^{\eta_1 + \bar{\eta}_2 + a_{12}} + e^{\eta_2 + \bar{\eta}_1 + a_{21}} + e^{\eta_2 + \bar{\eta}_2 + a_{22}}, \quad g^{(1)} = e^{\eta_1} + e^{\eta_2}, \quad (52)$$

$$f^{(2)} = e^{\eta_1 + \bar{\eta}_1 + \eta_2 + \bar{\eta}_2 + r}, \quad g^{(2)} = e^{\eta_1 + \bar{\eta}_1 + \bar{\eta}_2 + \delta_1} + e^{\eta_1 + \bar{\eta}_2 + \eta_2 + \delta_2}, \quad (53)$$

$$\begin{aligned} p_j^{(1)} = & \exp((k_1 + \bar{k}_1 - i\xi_j)x - \gamma_2(t) - \bar{\gamma}_2(t) + \gamma_1(t) + \bar{\gamma}_1(t) + \Omega_j(t) - r + a_{11}) + \\ & + \exp((k_1 + \bar{k}_2 - i\xi_j)x - \gamma_1(t) - \bar{\gamma}_1(t) + \gamma_2(t) + \bar{\gamma}_2(t) + \Omega_j(t) - r + a_{12}) + \\ & + \exp((k_2 + \bar{k}_1 - i\xi_j)x - \gamma_1(t) - \bar{\gamma}_2(t) + \bar{\gamma}_1(t) + \gamma_2(t) + \Omega_j(t) - r + a_{21}) + \\ & + \exp((k_2 + \bar{k}_2 - i\xi_j)x - \gamma_1(t) - \bar{\gamma}_1(t) + \gamma_2(t) + \bar{\gamma}_2(t) + \Omega_j(t) - r + a_{22}), \quad (54) \end{aligned}$$

$$p_j^{(2)} = \exp((k_1 + \bar{k}_1 + k_2 + \bar{k}_2 - i\xi_j)x + \Omega_j(t)), \quad (55)$$

$$\begin{aligned} h_j^{(1)} = & \frac{\exp((k_1 + \bar{k}_1 - i\xi_j)x - \gamma_2(t) - \bar{\gamma}_2(t) + \gamma_1(t) + \bar{\gamma}_1(t) + \Omega_j(t) - r)}{(k_1 + \bar{k}_1)(e^{\eta_1} + e^{\eta_2})} + \\ & + \frac{\exp((k_1 + \bar{k}_2 - i\xi_j)x - \gamma_2(t) - \bar{\gamma}_2(t) + \gamma_1(t) + \bar{\gamma}_1(t) + \Omega_j(t) - r)}{(k_1 + \bar{k}_2)(e^{\eta_1} + e^{\eta_2})} + \\ & + \frac{\exp((k_2 + \bar{k}_1 - i\xi_j)x - \gamma_2(t) - \bar{\gamma}_2(t) + \gamma_1(t) + \bar{\gamma}_1(t) + \Omega_j(t) - r)}{(k_2 + \bar{k}_1)(e^{\eta_1} + e^{\eta_2})} + \\ & + \frac{\exp((k_2 + \bar{k}_2 - i\xi_j)x - \gamma_2(t) - \bar{\gamma}_2(t) + \gamma_1(t) + \bar{\gamma}_1(t) + \Omega_j(t) - r)}{(k_2 + \bar{k}_2)(e^{\eta_1} + e^{\eta_2})}, \quad (j=1,2), \quad (56) \end{aligned}$$

here $\eta_j = k_j x + \gamma_j(t)$, ($j=1,2$), k_j are constants, $\gamma_j(t)$, $\Omega_j(t)$ ($j=1,2$) are continuous functions and

$$a_{mn} = \ln \frac{1}{(k_m + \bar{k}_n)^2}, \quad m, n = 1, 2$$

$$\delta_j = \ln \left(\frac{B_j}{q_j} \right), \quad j = 1, 2,$$

$$q_1 = (k_1 + \bar{k}_1)(k_2 + \bar{k}_1), \quad q_2 = (k_1 + \bar{k}_2)(k_2 + \bar{k}_2),$$

$$B_1 = \frac{(k_1 + \bar{k}_1 + k_2 + \bar{k}_2)}{(k_1 + \bar{k}_1)^2 (k_2 + \bar{k}_1)^2}, \quad B_2 = \frac{(k_1 + \bar{k}_1 + k_2 + \bar{k}_2)}{(k_1 + \bar{k}_2)^2 (k_2 + \bar{k}_2)^2},$$

$$r = \frac{(k_2 - k_1)^2 (\bar{k}_2 - \bar{k}_1)^2}{(k_1 + \bar{k}_1)^2 (k_1 + \bar{k}_2)^2 (k_2 + \bar{k}_1)^2}.$$

By applying functions (52)-(56) to equation (38), functions $\gamma_j(t)$, $\Omega_j(t)$ ($j=1,2$) and constants k_1 , k_2 can be found

$$\Omega_j(t) = \ln \beta_j(t) + \frac{1}{2}(\bar{\gamma}_j(t) - r + \delta_j) + \gamma_1(t) + \gamma_2(t), \quad (j=1,2), \quad (57)$$

$$\gamma_j(t) = -4i\bar{\xi}_j^2 t - 2 \int_0^t \bar{\beta}_j^2(\tau) d\tau + \gamma_j(0), \quad (j=1,2), \quad (58)$$

$$k_j = -2i\bar{\xi}_j, \quad (j=1,2). \quad (59)$$

Taking into consideration the above, we have proved the following theorem.

Theorem 3: If the function $u(x,t)$ in equation (34) satisfies the condition (36) and the functions $\varphi_{1j}(x,t)$, $\varphi_{2j}(x,t)$ in the system of equations (35) satisfy the condition (36`), then one soliton solution of the problem (34)-(35) is in the form (50) and two soliton solution is in the form of (51).

In the second paragraph of the third chapter, it was found soliton solution of the loaded nonlinear Schrodinger equation by using Hirota direct method.

We consider following

$$iu_t + 2|u|^2 u + u_{xx} + h(t)u_x = 0, \quad x \in R, \quad t \geq 0, \quad (60)$$

where $h(t) = -\gamma(t)u(0,t)$ and $u(x,t)$ is unknown function, $\gamma(t)$ - an arbitrary continuous function.

In this paragraph, $u(x,t)$ solution of the equation (60) following

$$\int_{-\infty}^{\infty} \left((1+|x|)|u(x,t)| + \sum_{j=1}^2 \left| \frac{\partial^j u(x,t)}{\partial x^j} \right| \right) dx < \infty, \quad t \geq 0 \quad (61)$$

is considered to exist in the class of functions and demanded to show the method of finding the solution.

We will find the soliton solution of the loaded nonlinear Schrödinger equation by using of Hirota direct method. With the help of the dependent variable transformations

$$u = \frac{g}{f}, \quad (62)$$

the equation (60) can be transformed into the bilinear forms

$$\begin{cases} iD_t g \cdot f + D_x^2 g \cdot f + h(t)D_x g \cdot f = 0, \\ D_x^2 f \cdot f = 2\bar{g}g, \end{cases} \quad (63)$$

where \bar{g} is the complex conjugation of the function g and Hirota's bilinear operators D_x and D_t are defined by

$$D_x^m D_t^n g(x,t) \cdot f(x,t) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'} \right)^n g(x,t) f(x',t') \Big|_{x=x', t=t'}.$$

We will give the analytical expression of one-soliton solution of the equation (60). According to in the known Hirota direct method, we consider for the one-soliton solution of (60) in the below form

$$f = 1 + \chi^2 f^{(1)}, \quad g = \chi g^{(1)}. \quad (64)$$

Substituting these equation into (63) and equating the coefficients of the same powers of χ , we have

$$g^{(1)} = e^{\xi_1}, \quad f^{(1)} = \frac{1}{(k_1 + \bar{k}_1)^2} e^{\xi_1 + \bar{\xi}_1}, \quad (65)$$

here $\xi_1 = k_1 x + \Omega_1(t)$, k_1 is constant and $\Omega_1(t)$ is an arbitrary function of t . We find the function $\Omega_1(t)$

$$\Omega_1(t) = ik_1^2 t + ik_1 \int_0^t \gamma(\tau) u(0, \tau) d\tau, \quad (66)$$

Thus, taking into account (64)-(66) and (62) we can write the one-soliton solution of loaded nonlinear Schrödinger equation in the following form

$$u(x, t) = \frac{e^{k_1 x + ik_1^2 t + ik_1 \int_0^t \gamma(\tau) u(0, \tau) d\tau}}{1 + \frac{1}{(k_1 + \bar{k}_1)^2} e^{(k_1 + \bar{k}_1)x + i(k_1^2 + \bar{k}_1^2)t + i(k_1 + \bar{k}_1) \int_0^t \gamma(\tau) u(0, \tau) d\tau}}, \quad (67)$$

We find two-soliton solution of the loaded nonlinear Schrödinger equation by following steps, but we get the functions f and g in the following form

$$g = \chi g^{(1)} + \chi^3 g^{(2)}, \quad f = 1 + \chi^2 f^{(1)} + \chi^4 f^{(2)}.$$

By applying the same previous procedure, we obtain the following set of equations from equations (3.2.3) corresponding to the different power of χ and by solving them we have

$$f^{(1)} = e^{\xi_1 + \bar{\xi}_1 + a_{11}} + e^{\xi_1 + \bar{\xi}_2 + a_{12}} + e^{\xi_2 + \bar{\xi}_1 + a_{21}} + e^{\xi_2 + \bar{\xi}_2 + a_{22}}, \quad g^{(1)} = e^{\xi_1} + e^{\xi_2}, \quad g^{(1)} = e^{\eta_1} + e^{\eta_2}, \quad (68)$$

$$f^{(2)} = e^{\xi_1 + \bar{\xi}_1 + \xi_2 + \bar{\xi}_2 + r}, \quad g^{(2)} = e^{\xi_1 + \bar{\xi}_1 + \xi_2 + \delta_1} + e^{\xi_1 + \xi_2 + \bar{\xi}_2 + \delta_2}, \quad (69)$$

where $\xi_j = k_j x + \Omega_j(t)$, $\Omega_j(t) = ik_j^2 t + ik_j \int_0^t \gamma(\tau) u(0, \tau) d\tau$, $j = 1, 2$, k_1 and k_2 are constans and

$$a_{mn} = \ln \frac{1}{(k_m + \bar{k}_n)^2}, \quad m, n = 1, 2$$

$$\delta_1 = \ln \left(\frac{1}{(k_1 + \bar{k}_1)^2} + \frac{1}{(k_2 + \bar{k}_1)^2} \right), \quad \delta_2 = \ln \left(\frac{1}{(k_2 + \bar{k}_2)^2} + \frac{1}{(k_1 + \bar{k}_2)^2} \right),$$

$$r = \frac{(k_2 + \bar{k}_2)^2 (k_2 + \bar{k}_1)^2 + (k_1 + \bar{k}_2)^2 (k_2 + \bar{k}_1)^2 + (k_1 + \bar{k}_2)^2 (k_2 + \bar{k}_2)^2}{(k_1 + \bar{k}_2)^2 (k_2 + \bar{k}_2)^2 (k_2 + \bar{k}_1)^2 (k_1 + \bar{k}_1 + k_2 + \bar{k}_2)^2}, \quad j = 1, 2,$$

Thus, taking into account (68)-(69) and (62) we can write the two-solution of equation (60) in the following form

$$u = \frac{g^{(1)} + g^{(2)}}{1 + f^{(1)} + f^{(2)}}, \quad (70)$$

Taking into consideration the above, we have proved the following theorem.

Theorem 4: If the function $u(x,t)$ in equation (60) satisfies the condition (61), then the one soliton solution of the equation (60) is in the form (67), two soliton solution is in the form of (70).

In the third paragraph of the third chapter it was shown to find the exact solution of loaded Korteweg-de Vries equation.

We consider the loaded Korteweg-de Vries equation

$$u_t - 6uu_x + u_{xxx} - \gamma(t)u(0,t)u_x = 0, \quad t \geq 0, \quad x \in \mathbb{R}, \quad (71)$$

where $\gamma(t)$ is an arbitrary given continuous function. We obtain three types of solutions of the traveling wave type of the equation (71) by using (G'/G) -expansion method. This is expressed in the following theorem.

Theorem 5: If $(\lambda^2 - 4\mu) > 0$, then the solution of equation (71) is in the form of hyperbolic functions

$$u(\xi) = \frac{k^2(\lambda^2 - 4\mu)}{2} \left(\frac{c_1 \operatorname{sh} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi + c_2 \operatorname{ch} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi}{c_1 \operatorname{ch} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi + c_2 \operatorname{sh} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi} \right)^2 - \frac{\lambda^2 k^2}{2} + 2k^2 \mu,$$

if $(\lambda^2 - 4\mu) < 0$, then the solution is in the form of trigonometric functions

$$u(\xi) = \frac{k^2(4\mu - \lambda^2)}{2} \left(\frac{-c_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi + c_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi}{c_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi + c_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi} \right)^2 - \frac{\lambda^2 k^2}{2} + 2k^2 \mu,$$

where $\xi = kx - k^3(\lambda^2 - 4\mu)t + k \int_0^t \gamma(\tau)u(0,\tau)d\tau + \Omega^0$; c_1 , c_2 and Ω^0 are arbitrary constants, if $(\lambda^2 - 4\mu) = 0$ then the solution is

$$u(x,t) = \frac{2k^2 c_2^2}{(c_1 + \xi c_2)^2},$$

where $\xi = kx + k \int_0^t \gamma(\tau)u(0,\tau)d\tau + \Omega^0$; c_1 , c_2 and Ω^0 are arbitrary constants.

In the fourth paragraph of the third chapter, it was shown to find the exact solution of loaded modified Korteweg-de Vries equation.

Consider the following loaded modified Korteweg-de Vries equation

$$q_t - 6q^2 q_x + q_{xxx} - \gamma(t)q(0,t)q_x = 0, \quad (72)$$

where $q(x,t)$ is an unknown function, $x \in \mathbb{R}$, $t \geq 0$, $\gamma(t)$ - is the given real continuous function. We obtain three types of solutions of the traveling wave type

of the equation (72) by using (G' / G) - expansion method. This is expressed in the following theorem.

Theorem 6: If $(\lambda^2 - 4\mu) > 0$, then the solution of equation (72) is in the form of hyperbolic functions

$$q(\xi) = \frac{k\sqrt{\lambda^2 - 4\mu}}{2} \left(\frac{c_1 \operatorname{sh} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi + c_2 \operatorname{ch} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi}{c_1 \operatorname{ch} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi + c_2 \operatorname{sh} \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi} \right),$$

where $\xi = kx + \frac{k^3(\lambda^2 - 4\mu)}{2}t + k \int_0^t \gamma(\tau)q(0, \tau)d\tau + \Omega^0$, c_1 , c_2 and Ω^0 are arbitrary constants, if $(\lambda^2 - 4\mu) < 0$, then the solution is in the form of trigonometric functions

$$q(\xi) = \frac{k^2(4\mu - \lambda^2)}{2} \left(\frac{-c_1 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi + c_2 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi}{c_1 \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi + c_2 \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi} \right)^2 - \frac{\lambda^2 k^2}{2} + 2k^2 \mu,$$

where $\xi = kx - k^3(\lambda^2 - 4\mu)t + k \int_0^t \gamma(\tau)u(0, \tau)d\tau + \Omega^0$; c_1 , c_2 and Ω^0 are arbitrary constants, if $(\lambda^2 - 4\mu) = 0$ then the solution is

$$q(\xi) = \frac{kc_2}{c_1 + \xi c_2},$$

where $\xi = kx + k \int_0^t \gamma(\tau)q(0, \tau)d\tau + \Omega^0$, c_1 , c_2 , and Ω^0 are arbitrary constants.

The author is sincerely grateful to his scientific supervisor, Dr. Urazboev Gayrat Urazalievich, for his constant attention and valuable advice in discussing the results of this dissertation.

CONCLUSIONS

The first chapter of the dissertation is devoted to inverse scattering theory for the Dirac operator and to solving nonlinear evolution equation by using direct methods.

The second chapter of the dissertation is dedicated to integrating loaded nonlinear Schrödinger equation via inverse scattering method and to find numerical solution of the Gelfand-Levitan-Marchenko integral equation.

The third chapter of the dissertation is concerned in finding the exact solution of the nonlinear Schrödinger equation with self-consistent source, loaded nonlinear Schrödinger equation, loaded Korteweg-de Vries equation and loaded modified Korteweg-de Vries equation.

The main results of the research are as follows:

- proved the complete integrability loaded nonlinear Schrödinger equation in the class of rapidly decreasing functions;
- proved the uniqueness theorem the Cauchy problems for the loaded nonlinear Schrödinger equation in the class of rapidly decreasing functions;
- shown numerical solutions of the system Gelfand-Levitan-Marchenko integral equations;
- found the soliton solutions of the nonlinear Schrödinger equation with self-consistent source in the class of rapidly decreasing functions;
- found the soliton solutions of the loaded nonlinear Schrödinger equation in the class of rapidly decreasing functions;
- shown exact solutions of the loaded Korteweg-de Vries equation
- shown exact solutions of the loaded modified Korteweg-de Vries equation.

**НАУЧНЫЙ СОВЕТ PhD.03/30.12.2019.FM.55.01
ПО ПРИСУЖДЕНИЮ УЧЕНОЙ СТЕПЕНИ ПРИ УРГЕНЧСКОМ
ГОСУДАРСТВЕННОМ УНИВЕРСИТЕТЕ**

УРГЕНЧСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

РАХИМОВ ИЛХАМ ДАВРОНБЕКОВИЧ

**ПРЯМЫЕ И ОБРАТНЫЕ МЕТОДЫ РЕШЕНИЯ НАГРУЖЕННЫХ
НЕЛИНЕЙНЫХ УРАВНЕНИЙ**

01.01.02 – Дифференциальные уравнения и математическая физика

**АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD)
ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ**

Ургенч – 2022

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Кабинете Министров Республики Узбекистан за B2022.2.PhD/FM710.

Диссертация выполнена в Ургенчском государственном университете.

Автореферат диссертации на трех языках (узбекский, английский, русский (резюме)) размещен на веб-странице Научного совета (www.ik-mat.urdu.uz) и на Информационно-образовательном портале «ZiyoNet» по адресу (www.ziynet.uz).

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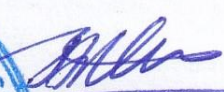
Ведущая организация: Самаркандский государственный университет

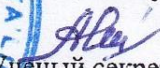
Защита диссертации состоится 24 сентября 2022 года в 14:00 часов на заседании Научного совета PhD.03/30.12.2019.FM.55.01 при Ургенчском государственном университете (Адрес: 220100, г. Ургенч, ул. Х. Алимджана, дом 14. Тел.: (99862)224-66-11, факс: (99862)224-67-00, e-mail: ik-mat.urdu@umail.uz).

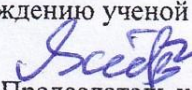
С диссертацией можно ознакомиться в Информационно-ресурсном центре Ургенчского государственного университета (зарегистрирована за № Д-618). (Адрес: 220100, г. Ургенч, ул. Х. Алимджана, дом 14. Тел.: (99862)224-66-11).

Автореферат диссертации разослан « 13 » сентября 2022 года
(протокол рассылки № 1 от « 13 » сентября 2022 года).




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ВВЕДЕНИЕ (аннотация диссертации доктора философии(PhD))

Целью исследования является исследование солитонных решений нелинейного уравнения Шредингера с самосогласованным источником, нагруженного нелинейного уравнения Шредингера, нагруженного уравнения Кортевега-де Фриза и нагруженного модифицированного уравнения Кортевега-де Фриза.

Объектами исследования являются нелинейное уравнение Шредингера с самосогласованным источником, нагруженное уравнение Кортевега-де Фриза, нагруженное модифицированное уравнение Кортевега-де Фриза, нагруженное нелинейное уравнение Шредингера, оператор Дирака.

Научная новизна исследовательской работы состоит в следующем:

доказана полная интегрируемость нагруженного нелинейного уравнения Шредингера в классе быстроубывающих функций;

доказана теорема единственности задачи Коши для нагруженного нелинейного уравнения Шредингера в классе быстроубывающих функций;

приведены численные решения системы интегральных уравнений Гельфанда-Левитана-Марченко;

найжены солитонные решения нелинейного уравнения Шредингера с самосогласованным источником в классе быстроубывающих функций;

найжены солитонные решения нагруженного нелинейного уравнения Шредингера в классе быстроубывающих функций;

показаны точные решения нагруженного уравнения Кортевега-де Фриза;

показаны точные решения нагруженного модифицированного уравнения Кортевега-де Фриза.

Внедрение результатов исследований. Полученные в диссертационной работе результаты были применены в следующих проектах:

- метод расчета основного интегрального уравнения теории обратного рассеяния численных решений системы интегральных уравнений Гельфанда-Левитана-Марченко был использован в проекте “Slovak Research and Development Agency under contract No. APVV-18-0308” и “Slovak Grant Agency VEGA No. 1/0358/20 and No. 2/0127/20”. (справка Университет имени Коменского в Братиславе, 24 июня 2022 г.);

- в проекте “Ordinary and Functional Differential Equations, code MTM2016-75140-P”, поддержанном Министерством экономики Испании в Университете Сантьяго-де-Компостела (Испания, справка Университета Сантьяго-де-Компостела, 27 июня 2022 г.), нелинейное уравнение Шредингера использовалось для нахождения численных аналитических решений.

Структура и объем диссертации. Диссертация состоит из трех глав, заключения и списка использованной литературы. Объем диссертации 81 страниц.

E'LON QILINGAN ISHLAR RO'YXATI
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ
LIST OF PUBLISHED WORKS

I bo'lim (Часть I; Part I)

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3. Urazboev G. U., A. A. Reyimbergganov, Rakhimov I. D. Numerical solution of the system of Marchenko integral equations. // Uzbek Mathematical Journal, 2021, vol. 65, no. 3, pp. 159-165. (01.00.00; №6)

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II bo'lim (Часть II; Part II)

9. Рахимов И. Д. Интегрирование нагруженного нелинейное уравнение Шредингер. // Глобаллашув даврида математика ва амалий математиканинг долзарб масалалари, Республика илмий анжумани, Ташкент, 1-2 июн, 2021, б. 364-365.

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11. Urazboev G.U., Reyimbergganov A.A., Rakhimov I.D. Numerical solution of the system of Marchenko integral equations. // Abstracts of the international

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Автореферат Урганч давлат университети ноширлик бўлимида таҳрирдан
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